Vertical Collusion *

By David Gilo[†] and Yaron Yehezkel[‡]

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Abstract

We characterize the features of collusion involving retailers and their supplier, who engage in secret vertical contracts and all equally care about future profits ("vertical collusion"). We show such collusion is easier to sustain than collusion among retailers. Furthermore, vertical collusion can solve the supplier's inability to commit to charging the monopoly wholesale price when retailers are differentiated. The supplier pays retailers slotting allowances as a prize for adhering to the collusive scheme and rejects deviations from the collusive vertical contract. In the presence of competing suppliers, vertical collusion can be sustained using short – term exclusive dealing in every period with the same supplier, if the supplier can inform a retailer that the other retailer did not offer the supplier exclusivity.

Keywords: vertical relations, tacit collusion, exclusive dealing, opportunism, slotting allowances.

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[†] Buchmann Faculty of Law, Tel Aviv University (email: gilod@tauex.tau.ac.il)

[‡] Coller School of Management, Tel Aviv University (email: yehezkel@tauex.tau.ac.il)

1 Introduction

This paper asks what are the features of ongoing collusion involving not only retailers, but also their joint supplier (all of whom are strategic players caring about future profits), and whether such collusion is more sustainable than collusion among retailers that does not involve a forward-looking supplier. Retailers (or other intermediaries) would prefer to collude at the expense of consumers, but competition among them is often too intense to support such collusion. Retailers typically buy from a joint supplier, where all firms interact repeatedly. The supplier is typically a strategic player too, who, like retailers, cares about future profits. This raises the question: can including the supplier in the collusive scheme improve the prospects of collusion, and if so, how?

We consider an infinitely repeated game involving competing retailers and a joint supplier (we later extend the model to multiple suppliers). In every period, retailers offer secret, one-period two-part tariff contracts to the supplier, and then play a game of incomplete information by setting retail prices without observing the contract offer their rival made to the supplier. All three firms have the same discount factor, so that retailers cannot rely on a more patient supplier to assist them in colluding. Since vertical contracts are secret, retailers cannot use observable vertical contracts as a commitment device in order to raise the retail price.[1]

We find that the retailers and the supplier can engage in a collusive scheme involving all of them. We refer to such a scheme as "vertical collusion". Each of the three firms has a short-run incentive to deviate from collusion and increase its own current-period profit at the expense of the other two, yet they collude because they all gain a share of future collusive profits, should they adhere to the collusive scheme in the current period. The three firms manage to do so even when retailers are too short-sighted to maintain standard horizontal collusion between themselves. Hence vertical collusion is easier to sustain than horizontal collusion.

The collusive mechanism works as follows. In every period, each retailer asks the supplier to pay the retailer a fixed fee. The fixed fee implicitly rewards the retailer for adhering to the collusive price in the previous period, and increases the retailers' future

[1]Importantly, exchange of information among retailers competing in a downstream market regarding the terms of their contracts with a supplier is likely to be an antitrust violation. See Department of Justice/Federal Trade Commission (2000); European Commission (2011); Federal Trade Commission (2011); OECD (2010) and New Zealand Commerce Commission (2014).

gains from collusion. Retailers expect that the supplier will continue rewarding them in the future only if they maintain the collusive scheme. The supplier, for his part, does not agree to pay retailers fixed fees unless they offer him a higher wholesale price than the one he would receive absent collusion: The supplier expects that retailers will continue rewarding him in the future with a high wholesale price only if he maintains the collusive scheme. Hence, the collusive mechanism involves transferring some of the retailers' collusive profit in a current period to the supplier (through higher wholesale prices payed to the supplier), and receiving part of these profits back in the future, conditional on colluding in the current period. The higher wholesale price too has repercussions on the retailers' incentives to collude: While it lowers their profits from deviating from collusion, it also lowers their profits from colluding. We derive a general condition under which these repercussions either further increase, or do not substantially decrease, the retailers' incentive to collude, and show that this condition holds at least when retailers are close substitutes.

For vertical collusion to be sustainable, parties need to be induced not to deviate from these collusive vertical contracts. This is challenging when vertical contracts are secret, because one retailer does not observe whether the other retailer made a side-deal with the supplier in order to deviate from collusion. We show that when a retailer attempts to deviate from the collusive price by offering the supplier a different vertical contract without compensating the supplier, the supplier rejects the retailer's offer and refuses to sell him the product. Having to compensate the supplier to avoid such rejection renders the retailer's deviation unprofitable.

We then extend the analysis to the case of multiple suppliers competing over selling a homogeneous product to homogeneous retailers. We show that there is a vertical collusion equilibrium in which retailers endogenously offer, in every period, to buy exclusively from the same supplier. Hence the collusive equilibrium is sustained with single-period exclusive dealing commitments. The supplier is induced to assist this vertical collusive scheme, because otherwise he makes no profits, due to intense competition from other suppliers. We assume a retailer can renegotiate the contract when the supplier informed him, in the form of cheap talk, that the competing retailer did not offer to buy exclusively from the supplier. We show that the supplier is induced to reveal the truth.

Our results have several policy implications. In particular, we identify practices that may have the potential, in appropriate market circumstances, to be harmful to competition. Our results can be used as a factor that can shed new light on the antitrust treatment of these practices, and that can be balanced against their possible virtues.

First, the paper sheds a new light on exclusive dealing arrangements, where a retailer promises to buy from a single supplier. We show that exclusive dealing agreements between buyers and one of the suppliers may facilitate vertical collusion even when the promise to deal exclusively with the supplier is for only a short term. This result stands in stark contrast to current antitrust rulings. Antitrust courts and agencies hold that exclusive dealing contracts that bind a buyer to a supplier for only a short term are automatically legal, and such soft antitrust treatment is also advocated by the antitrust literature.[2] We show that with repeated interaction between a supplier and his customers, exclusive dealing may become a self-enforcing practice. In each period, each of the retailers binds himself to the same supplier for only this period. It is the collusive equilibrium, however, that induces all retailers to offer to buy only from this supplier in subsequent periods as well.

A second, related, policy implication of our results is that antitrust courts and agencies should, in appropriate cases, be stricter toward a supplier that shares information with a retailer on whether a competing retailer offered him exclusivity. Such antitrust scrutiny can cause the vertical collusive scheme to break down in the presence of competition among suppliers.

The third policy implication is with regard to the antitrust treatment of a supplier's refusal to deal with a retailer. Our analysis shows that the supplier's ability to unilaterally refuse a deviating retailer's offer plays a key role in the sustainability of vertical collusion. By contrast, US case law takes a soft approach toward a supplier's refusal to deal with retailers that do not adhere to the supplier's policy regarding prevention of price competition among retailers over the supplier's brand.

Finally, our paper shows that slotting allowances (fixed fees often paid by suppliers to retailers in exchange for shelf space, promotional activities, and the like) may be more anticompetitive than currently believed. In our framework, slotting allowances facilitate the vertical collusion scheme even though vertical contracts are secret. Current literature implies that such practices can facilitate downstream collusion only when vertical con-

[2] See, e.g., Areeda and Hovenkamp (2011a).

tracts are observable. Hence slotting allowances with a supplier selling a strong brand, or with a supplier with whom retailers deal exclusively, deserve stricter antitrust treatment than currently believed.[3]

Our paper is related to several strands of the economic literature. The first strand involves vertical relations in a repeated infinite horizon game. Asker and Bar-Isaac (2014) show that an incumbent supplier can exclude the entry of a forward-looking entrant by offering forward-looking retailers, on an ongoing basis, part of the incumbent's monopoly profits, via vertical practices such as resale price maintenance, slotting fees, and exclusive territories. Because retailers in their model care about future profits, they may prefer to keep a new supplier out of the market, so as to continue receiving a portion of the incumbent supplier's profits. While their paper focuses on the importance of retailers being forward looking so that they can help a monopolistic supplier entrench his monopoly position, our paper focuses on the importance of the supplier being forward looking so as to enable a tacitly collusive retail price. Another part of this literature examines collusion among retailers, where suppliers are myopic. In particular, Normann (2009) and Nocke and White (2010) find that vertical integration can facilitate downstream collusion between a vertically integrated retailer and independent retailers. In a paper closely related to ours, Piccolo and Miklós-Thal (2012) show that retailers with bargaining power can collude by offering myopic suppliers a high wholesale price and negative fixed fees. They were the first to show how high wholesale prices, combined with slotting allowances, can help sustain ongoing collusion among retailers. Our paper contributes to theirs in that they assume the retailers observe each other's vertical contracts with the supplier before they set the retail price, and this is what deters them from deviating from the collusive vertical contract. We contribute to this idea by assuming retailers cannot observe each other's vertical contract and showing how a joint forward-looking supplier can replace the role of observability by each retailer of his rival's vertical contract: in our model, what deters retailers from deviating from the collusive contract is that the supplier will reject the deviation and refuse to deal with the deviating retailer, and compensating the supplier so as to cause him not to reject renders the deviation unprofitable. Second,

[3] At the same time, Chu (1992), Lariviere and Padmanabhan (1997), Desai (2000) and Yehezkel (2014) show that slotting allowances may also have the welfare enhancing effect of enabling suppliers to convey information to retailers concerning demand. See also Federal Trade Commission (2001, 2003), and European Commission (2012).

Piccolo and Miklós-Thal also consider information exchange. They show that retailers have an incentive to credibly reveal to each other their vertical contracts, as doing so facilitates collusion. We contribute to this idea by considering a joint forward-looking supplier, who has an incentive to police adherence to the collusive scheme even when retailers do not share such information about each other. Furthermore, in section 6, where we discuss multiple suppliers, we show how communication between the supplier and each retailer about whether the competing retailer offered the supplier an exclusive dealing contract can facilitate collusion. Hence, while Piccolo and Miklós-Thal consider the case where each retailer deals with a separate supplier, we analyze the case where all retailers deal with the same supplier, either because there is only one supplier, or because retailers commit to buying only from one of the suppliers.

Doyle and Han (2012) consider retailers that can sustain downstream collusion by forming a buyer group that jointly offers contracts to myopic suppliers. The rest of this literature studies collusion among suppliers, where retailers are myopic: Jullien and Rey (2007) consider an infinite horizon model with competing suppliers where each supplier sells to a different retailer and offers it a secret contract. Their paper studies how suppliers can use resale price maintenance to facilitate collusion among the suppliers, in the presence of stochastic demand shocks. Nocke and White (2007) consider collusion among upstream firms and the effect vertical integration has on such collusion. Reisinger and Thomes (2015) analyze a repeated game between two competing and long-lived manufacturers that have secret contracts with myopic retailers. They find that colluding through independent, competing retailers is easier to sustain and more profitable to the manufacturers than colluding through a joint retailer. Schinkel, Tuinstra and Rüggeberg (2007) consider collusion among suppliers in which suppliers can forward some of the collusive profits to downstream firms in order to avoid private damages claims. Piccolo and Reisinger (2011) find that exclusive territories agreements between suppliers and retailers can facilitate collusion among suppliers. The main difference between our paper and this literature is that we examine collusion involving the whole vertical chain: supplier and retailers alike, who are all forward looking, and all have a short run incentive to deviate from collusion which is balanced against a long run incentive to maintain the collusive equilibrium. We show that additional strategic considerations come into play when both the retailers and the supplier are forward-looking players.

The second strand of the literature concerns static games in which vertical contracts serve as a device for reducing price competition between retailers. Bonanno and Vickers (1988) find that suppliers can use two-part tariffs that include a wholesale price above marginal cost in order to relax downstream competition, and a positive fixed fee, to collect the retailers' profits. In Shaffer (1991) and (2005), Innes and Hamilton (2006), Rey, Miklós-Thal and Vergé (2011) and Rey and Whinston (2012), retailers have buyer power, hence two-part tariffs involve the suppliers paying fixed fees to retailers.

The difference between our paper and this strand of the literature is that we study a repeated game rather than a static game. This enables us to introduce the concept of vertical collusion, where the supplier, as well as retailers, care about future profits. Also, in this literature, vertical contracts are observable to retailers. We consider the prevalent case where vertical contracts are unobservable.

The third strand of literature involves static vertical relations in which a supplier behaves opportunistically by granting price concessions to one retailer at the expense of the other. Hart and Tirole (1990), O'Brien and Shaffer (1992), McAfee and Schwartz (1994) and Rey and Vergé (2004) consider suppliers that make secret contract offers to retailers. They find that a supplier may behave opportunistically and offer secret discounts to retailers. Anticipating this, retailers do not agree to pay high wholesale prices. The vertical collusive scheme we identify resolves an opportunism problem similar to the one exposed in the above literature and restores the supplier's power to charge high wholesale prices.

2 The model

Consider two downstream retailers, R_1 and R_2 that compete in prices and a joint upstream supplier. Suppose that retailers are horizontally differentiated. The demand function facing R_i is $q(p_i, p_j)$, where R_i charges the price p_i, and $q(p_i, p_j)$ is decreasing with p_i and increasing with p_j. The demand function when only R_i sells the product is $\widehat{q}(p_i) \equiv q(p_i, \infty)$. All firms' costs are zero.

Let p_M denote the monopoly price that maximizes $p_i q(p_i, p_j) + p_j q(p_j, p_i)$ with respect to p_i. The monopolistic quantity of each retailer is $q_M \equiv q(p_M, p_M)$. As retailers are imperfect substitutes, we assume that if only one retailer monopolizes the market, the demand it faces is smaller than retailers' aggregate demand when they both operate and charge the monopoly price: $q_M \leq \widehat{q}(p_M) \leq 2q_M$, where the first (second) inequality becomes an equality when retailers are fully differentiated (homogeneous).

The retailers' Nash competitive prices are defined as follows. Let $p(w_i; p_j)$ denote R_i's price that maximizes $(p_i - w_i)q(p_i, p_j)$ with respect to p_i, given that R_i pays a wholesale price of w_i and expects that R_j charges p_j. Notice that $p(w_i; p_j)$ is increasing with w_i. Let $p(w_i, w_j)$ denote R_i's Nash equilibrium price given that both retailers play their $p(w_i; p_j)$.

The two retailers and the supplier interact for an infinite number of periods and have a discount factor, δ, where $0 \leq \delta \leq 1$. The timing of each period is as follows. In the first stage, retailers offer a take-it-or-leave-it contract to the supplier (simultaneously and non-cooperatively). Each R_i offers a contract (w_i, T_i), where w_i is the wholesale price and T_i is a fixed payment from R_i to the supplier that can be positive or negative. In the latter case the supplier pays slotting allowances to R_i. The supplier observes the offers and decides whether to accept one, both or none. All of the features of the bilateral contracting between R_i and the supplier are unobservable to R_j, $j \neq i$, throughout the game. Moreover, R_j cannot know whether R_i signed a contract with the supplier until the end of the period, when retail prices are observable. The contract offer is valid for the current period only.[4]

In the second stage of each period, the two retailers set their retail prices for the current period, p_1 and p_2, simultaneously and non-cooperatively. That is, retailers set prices after agreeing with the supplier on the fixed fees for the current period. At the end of the stage, retail prices become common knowledge (but again retailers cannot observe the contract offers).

We consider pure-strategy, perfect Bayesian-Nash equilibria. We focus on symmetric equilibria, in which along the equilibrium path both retailers choose the same strategy.

When there is no upstream supplier and the product is available to retailers at marginal cost (i.e., $w_i = w_j = 0$), retailers only play the second stage in every pe-

[4]See Piercy (2009), claiming that large supermarket chains in the UK often change contractual terms, including the wholesale price and slotting allowances, on a regular basis, e.g., via e-mail correspondence; Lindgreen, Hingley and Vanhamme (2009), discussing evidence from suppliers according to which large supermarket chains deal with them without written contracts and with changing price terms; See also "How Suppliers Get the Sharp End of Supermarkets' Hard Sell, The Guardian, http://www.theguardian.com/business/2007/aug/25/supermarkets.

riod, in which they decide on retail prices, and therefore the game consists of standard infinitely-repeated price competition between two differentiated firms. Then, a standard result is that horizontal collusion over the monopoly price is possible if:

$$\frac{p_M q_M}{1-\delta} \geq p(0;p_M)q(p(0;p_M),p_M) + \frac{\delta}{1-\delta}\pi_R^C, \tag{1}$$

where $\pi_R^C \equiv p(0,0)q(p(0,0),p(0,0))$ is the retailer's Nash equilibrium profit given that $w_i = w_j = 0$. The left hand side is the retailer's sum of infinite discounted profit from colluding on the monopoly price and the right hand side is the retailer's profit from deviating to $p(0;p_M)$ in the current period, followed by the Nash equilibrium profit in all future periods. Hence, collusion is possible if $\delta \leq \delta^C$, where:

$$\delta^C \equiv \frac{p(0;p_M)q(p(0;p_M),p_M) - p_M q_M}{p(0;p_M)q(p(0;p_M),p_M) - \pi_R^C}. \tag{2}$$

Notice that when retailers are close substitutes, $p(0;p_M) \to p_M$, $q(p(0;p_M),p_M) \to 2q_M$ and $\pi_R^C \to 0$, hence $\delta^C \to \frac{1}{2}$. Given this benchmark value of δ^C, we ask whether vertical collusion is sustainable for $\delta < \delta^C$. That is, whether retailers and their supplier can use vertical contracts, (w_i, T_i), to help sustain collusion when horizontal collusion is not sustainable.

3 Competitive static equilibrium benchmark

This section derives a competitive equilibrium benchmark in which the three firms expect that the outcome of the current period has no effect on the future. In the next section, we will assume that an observable deviation from vertical collusion will result in playing the competitive equilibrium in all future periods.

Consider a symmetric equilibrium with the following features. In stage 1, both retailers offer the contract $\left(T^C, w^C\right)$ that the supplier accepts. Then, in stage 2, both retailers set p^C and equally split the market. Each retailer earns $(p^C - w^C)q(p^C,p^C) - T^C$ and the supplier earns $2w^C q(p^C,p^C) + 2T^C$.

Following O'Brien and Shaffer (1992), we first show that when retailers are differentiated – in the sense that retailers can gain a positive profit margin – the monopoly outcome cannot be an equilibrium. This is established in the following lemma (all proofs are in the Appendix):

Lemma 1. *When retailers are differentiated, retailers cannot implement the monopoly outcome in any static equilibrium.*

Intuitively, the supplier and R_i agree on a contract that maximizes their joint profit given R_j's equilibrium contract. With differentiation, R_j gains a positive profit margin: $p(w^C; p_i) > w^C$, which R_i and the supplier do not internalize. Hence, they have an incentive to behave opportunistically and undercut p_i below the monopoly price. This enables them to make a profit at the expense of R_j. To do so, they agree on a wholesale price lower than the wholesale price R_j is paying. This causes monopoly pricing to collapse as an equilibrium. Notice that this result holds regardless of the extent to which retailers are differentiated. It only requires that retailers have a positive profit margin.

As O'Brien and Shaffer (1992) show, the same logic applies to any $w^C > 0$. In any putative equilibrium in which $w^C > 0$, the supplier and R_i will have an incentive to behave opportunistically and undercut w^C, because they don't internalize R_j's profit margin. This leaves $w^C = 0$ (and consequently $T^C = 0$) as the only pure strategy equilibrium.[5]

In what follows, we focus on this unique pure strategy equilibrium of the static game, in which, $w^C = 0$ and $T^C = 0$. Retailers' profits in the static equilibrium are $\pi_R^C = p(0,0)q(p(0,0), p(0,0))$ and the supplier earns 0.[6]

4 Collusive equilibrium

This section derives conditions under which vertical collusion can be sustainable for $\delta < \delta^C$. As shown below, these conditions always hold when retailers are close substitutes, and may also hold, depending on the features of the market, for any differentiation level.

Consider a collusive equilibrium with the following features. In the first stage of every period, both retailers offer the same collusive contract, (w^*, T^*) that the supplier accepts.

[5] For a pure strategy equilibrium to exist, R_i needs to believe that, given $w^C = 0$, R_i cannot motivate the supplier to reject R_j's offer by offering a contract with $w_i > 0$. If R_i believes that he can convince the supplier to reject R_j's offer, then no pure strategy equilibrium exists.

[6] Our qualitative results continue to hold when the static equilibrium involves a positive profit for the supplier (which may occur in a mixed strategy equilibrium). In this case too vertical collusion is still needed in order to achieve monopoly pricing, since by lemma 1, the industry profit is lower than the monopoly profit.

Then, in stage 2, both retailers set the monopoly price, p_M. Given an equilibrium contract, (w^*, T^*), in every period each retailer earns $\pi_R(w^*, T^*) = (p_M - w^*)\, q_M - T^*$ and the supplier earns $\pi_S\,(w^*, T^*) = 2(w^* q_M + T^*)$. Suppose that whenever a publicly observable deviation occurs (i.e., a retailer sets a price different than p_M or does not carry the product), retailers play the competitive equilibrium defined in section 3 in all future periods.

We look at a marginal decrease in δ below δ^C and show how retailers and the supplier can sustain vertical collusion with a contract that involves $T^* < 0$ and $w^* > 0$.

Suppose retailers offered the supplier a contract (w^*, T^*) that the supplier accepted. The collusive contract has to satisfy the following conditions. First, R_i needs to be induced to charge the monopoly price p_M in stage 2 rather than deviating to $p(w; p_M)$.[7] When retailers collude, R_i earns $(p_M - w)q_M - T$ in this and all future periods. If R_i deviates to $p(w; p_M)$, R_i gains a higher demand in the current period, $q(p(w; p_M), p_M) > q_M$, but collusion stops and R_i earns π_R^C in all future periods. R_i will not deviate from collusion if:

$$\frac{(p_M - w)\, q_M - T}{1 - \delta} \geq (p(w; p_M) - w) q(p(w; p_M), p_M) - T + \frac{\delta}{1 - \delta} \pi_R^C. \tag{3}$$

Next consider the supplier's incentive constraint. The supplier needs to be incentivized not to deviate from the collusive scheme by accepting only one of the retailers' offers thereby paying slotting allowances only once. If the supplier accepts both offers, collusion follows to the next period and the supplier earns $2(wq_M + T)$ in every period. If the supplier rejects R_i's offer, R_j can detect this deviation only at the end of stage 2, when R_j observes that R_i didn't offer the product. Therefore, in stage 2 R_j will still charge the monopoly price p_M and sell $\widehat{q}(p_M)$, implying that in the current period the supplier earns $w\widehat{q}(p_M) + T$ and collusion breaks down in all future periods, in which the supplier earns 0. Accordingly, the supplier will not deviate if:

$$\frac{2(wq_M + T)}{1 - \delta} \geq w\widehat{q}(p_M) + T. \tag{4}$$

Condition (3) imposes a ceiling on T while condition (4) imposes a floor on T. Let $T_R(w, \delta)$ and $T_S(w, \delta)$ denote the T that solves (3) and (4) in equality, respectively.

[7] To simplify notation, we will omit the "*" unless necessary.

Combining the two conditions, we have that a collusive contract, (w, T), has to satisfy: $T_S(w, \delta) \leq T \leq T_R(w, \delta)$.

To illustrate how vertical collusion is possible for $\delta < \delta^C$, we proceed in the following steps, illustrated in Figure 1. First, we show that in order to ensure that retailers collude when $\delta < \delta^C$, it must be that $T^* < 0$ (that is, the supplier must pay retailers slotting allowances). In the second step we show that if the supplier must pay slotting allowances, w^* must be sufficiently high, because otherwise the supplier will not want to participate in the collusive scheme. In the third step, we show that a higher w^* may then have repercussions for the retailers' incentive to collude.

[Figure 1 here]

Let us establish these results: The first step is to show the importance of slotting allowances for the collusive scheme. The following lemma shows that slotting allowances (a negative T) are required for retailers not to deviate. Slotting allowances serve implicitly as a prize the supplier pays retailers in future periods for them adhering to the collusive scheme in the current period.

Lemma 2. ***(For δ slightly below δ^C and w equal to or slightly above zero, vertical collusion requires slotting allowances:)*** *$T_R(0, \delta^C) = 0$ and $T_R(0, \delta)$ is increasing with δ.*

Lemma 2 implies that when the wholesale price is zero and the discount factor is equal to the critical discount factor, no fixed fees are needed to relax retailers' incentive constraint. For a small w and starting from $\delta = \delta^C$, as retailers become more impatient, negative fixed fees, i.e., slotting allowances, are needed to deter them from deviating. Since $T_R(0, \delta)$ is increasing with δ (and decreasing when δ declines) the level of slotting allowances required to relax retailer's incentive constraint increase as retailers become more impatient (i.e., $T_R(0, \delta)$ becomes more negative as δ decreases). Note that slotting allowances increase only the future collusive profits of retailers and have no effect on their profit from deviation. This is because in the current period, a retailer receives a slotting allowance whether or not he deviated. In equilibrium, each retailer expects that by setting p_M in the current period, the supplier will implicitly "reward" the retailer in the next periods by paying him slotting allowances.

The second step is to show that for the supplier to be willing to pay retailers slotting allowances and participate in the collusive scheme, the wholesale price they pay him needs to be positive. The supplier is willing to aid the collusive scheme since he can use it to charge a positive wholesale price. In particular, the supplier needs to solve his own opportunism problem that prevents him from making profits in the static game:

Lemma 3. ***(A positive w is necessary for the supplier to participate in the collusive scheme:)*** *$T_S(0, \delta) = 0$ for all δ and $T_S(w, \delta)$ is decreasing with w.*

The third step is to notice is that the increase in w has repercussions regarding the retailers' incentive to collude through condition (3). This effect can be positive or negative, because an increase in w decreases both the retailers' profit from maintaining collusion and their profit when deviating from collusion.

The following proposition provides the conditions for vertical collusion to be sustainable:

Proposition 1. ***(Necessary condition for sustainability of vertical collusion for δ slightly below δ^C):*** *A necessary condition for $T_R(w, \delta) \geq T_S(w, \delta)$ (sustainability of vertical collusion given the vertical contract) when $\delta < \delta^C$ is:*

$$\left.\frac{\partial(T_R(w, \delta) - T_S(w, \delta))}{\partial w}\right|_{w=0,\delta=\delta^C} = \tag{5}$$

$$\frac{1-\delta^C}{\delta^C}\left[q(p(0; p_M), p_M) - \frac{q_M}{(1-\delta^C)}\right] - \frac{(1-\delta^C)}{1+\delta^C}\left[\widehat{q}(p_M) - \frac{2q_M}{1-\delta^C}\right] > 0.$$

Furthermore, if (5) holds, then for δ marginally below δ^C, there is a cutoff, w^*, such that $T_R(w, \delta) \geq T_S(w, \delta)$ if $w \geq w^*$. Finally, $w^* \to 0^+$ as $\delta \to \delta^{C-}$.

Under condition (5) vertical collusion is possible for a δ slightly lower than δ^C, because then a marginal increase in w above $w = 0$ creates a gap between $T_R(w, \delta)$ and $T_S(w, \delta)$. With such a gap, there exists a pair (w, T) that satisfies: $T_S(w, \delta) \leq T \leq T_R(w, \delta)$. The first term in (5) is the effect of a marginal increase in w on the retailers' incentive to collude, and the second term is the effect of a marginal increase in w on the supplier's incentive to deviate from collusion.

To see the intuition, recall from (3) that as w increases, the retailer's profits from both maintaining collusion and deviating from collusion decrease. The retailer's profit from deviating from collusion decreases by the deviating quantity, $q(p(w;p_M),p_M)$, and his profit from maintaining collusion decreases by the monopoly quantity in all future periods, $\frac{q_M}{1-\delta}$ (the effect on retailers' incentive constraint is represented by the first term in (5)). If the effect of w on the profit from deviation is higher than the effect of w on the profit from collusion, an increase in w increases the retailer's incentive to collude. Intuitively, when retailers are sufficiently close substitutes, even a small deviation by R_i from collusion substantially increases R_i's demand, which makes the deviation less profitable the higher is w. This gives the parties more leeway to sustain vertical collusion. At the extreme, with homogenous retailers, vertical collusion becomes sustainable no matter how low δ is (as we will show in the next section). When retailers are sufficiently differentiated, however, a retailer's profit from deviation is limited, since it cannot steal the entire market share of the competing retailer by only slightly undercutting the collusive price. Then, an increase in w may encourage retailers to deviate. Indeed, when retailers are sufficiently differentiated, the first term in (5) can be negative.

Turning to the supplier's incentive constraint, it follows from (4) that any increase in w further deters the supplier from deviating from the collusive scheme (this effect is also manifested in the second term in (5)). An increase in w increases the supplier's incentive to deviate from collusion by the monopoly quantity of a single retailer, $\widehat{q}(p_M)$, and increases the supplier's incentive to participate in collusion by the total collusive quantity in all periods, $\frac{2q_M}{1-\delta}$. Since $\widehat{q}(p_M) \leq 2q_M$, an increase in w always decreases the supplier's incentive to deviate from collusion. Accordingly, the second term in (5) is always positive. The higher is the level of differentiation, the larger the gap between $\frac{2q_M}{1-\delta}$ and $\widehat{q}(p_M)$ and the more an increase in w helps induce the supplier to participate in vertical collusion. Intuitively, with high differentiation, the supplier benefits more from vertical collusion (gaining a high wholesale price on the larger demand for both retailers' outlets) and gains less from deviating from it (thereby selling only through one retailer and losing the potential demand loyal to the excluded retailer).

Since the second term in (5) is positive and the first term is positive if retailers are sufficiently close substitutes, we have that this condition always holds if retailers are sufficiently close substitutes, and can also hold if retailers are differentiated, as long as the first term is either positive or not too negative. Moreover, the higher is (5), even a lower w^* is sufficient to satisfy $T_S(w,\delta) \leq T \leq T_R(w,\delta)$. Whether this condition holds for any degree of retailer differentiation depends on market conditions.[8]

The remaining requirement is that in stage 1, R_i does not find it profitable to deviate to any other contract, $(w_i, T_i) \neq (w^*, T^*)$. In what follows, we show that the supplier is the key to prevent such deviations. Any contract deviation that enables R_i to deviate from collusion and steal R_j's market share makes the supplier lose on his contract with R_j. Hence, absent compensation from R_i, the supplier rejects any contract deviation by R_i and refuses to supply R_i the product. To convince the supplier to accept the deviation, R_i needs to compensate the supplier accordingly. But this, in turn, renders the contract deviation unprofitable to R_i. Hence, although R_j cannot see R_i's contract deviation, the supplier, of course, sees it, and prevents it. This mechanism is different than in Piccolo and Miklós-Thal (2012), where contracts are observable, and what deters R_i from making a contract deviation is that R_j can observe the deviation in the current period.

The benefit to R_i and the supplier from a contract deviation depends on their out-of-equilibrium beliefs concerning each other's future strategies given the deviation. In particular, when deciding whether to accept the deviating contract, the supplier needs to form beliefs on whether this contract will motivate R_i to deviate from collusion. Likewise, if the supplier accepts the deviating contract, R_i needs to form beliefs on whether the supplier accepts R_j's equilibrium offer. Suppose beliefs are "rational" in the sense that whenever R_i offers a deviating contract, R_i and the supplier correctly anticipate each-others' unobservable and rational response to this deviation. More precisely, consider a deviation by R_i to a contract $(w_i, T_i) \neq (w^*, T^*)$. Any such (w_i, T_i) can either cause collusion to stop or cause it to continue in future periods. We assume that both R_i and the supplier understand whether the deviation will cause collusion to stop or not.

The case of interest is the first option, where R_i and the supplier believe the contract deviation, $(w_i, T_i) \neq (w^*, T^*)$, will cause collusion to stop. In the period of any such deviation, R_i earns $(p(w_i; p_M) - w_i)q(p(w_i; p_M), p_M) - T_i$, but needs to compensate the supplier for his alternative profit from rejecting the deviation, accepting R_j's equilibrium contract and earning $w^*\widehat{q}(p^M) + T^*$. The following lemma shows that when w^* is small, condition (3) is sufficient to make such a deviation unprofitable for R_i:

[8] In an online note (available at: `https://www.tau.ac.il/~yehezkel/`), we offer an example in which retailers are almost fully differentiated such that the first term is negative. Yet, the above condition holds and there is a collusive equilibrium for $\delta > \delta^*$, where $0 < \delta^* < \delta^C$.

Lemma 4. *When w^* is not too high (sufficiently close to 0), (3) also ensures that R_i will not deviate to any contract that motivates R_i to defect from collusion.*

Recall that condition (3) ensures that R_i does not want to deviate from the collusive price. Lemma 4 shows that at least when w^* is sufficiently small, (3) also ensures that R_i will not deviate to any contract that motivates R_i to defect from collusion. Intuitively, deviating from both the collusive contract and collusion is less profitable for R_i than deviating from collusion without deviating from the collusive contract because in the latter deviation R_i needs to compensate the supplier for the supplier's loss of revenues from R_j.[9]

Lemma 4 and Proposition 1, taken together, show that when δ is slightly below δ^C and (5) holds, a marginal increase in w^* makes it possible to satisfy the retailers' and the supplier's incentive constraints given the collusive contract (Proposition 1). The contract also deters any retailer from making a contract deviation that stops collusion at least when w^* is small enough (lemma 4).

5 Vertical collusion with homogeneous retailers

In this section we consider the special case in which retailers are homogeneous. With this simplifying assumption we can study how δ affects the collusive contract and the way retailers share the collusive profit with the supplier. Since the collusive scheme in the presence of retailer differentiation converges to the homogeneous retailer case as retailers becomes close substitutes, the homogeneous case provides a tractable benchmark for examining vertical collusion with differentiated retailers.

Suppose that the two retailers are homogeneous and face the demand $Q(p)$. As before, let p_M denote the monopoly price that maximizes $pQ(p)$ and let $Q_M = Q(p_M)$. This is a special case of the previous section with $2q_M = Q_M$. Recall that with homogeneous retailers, $\delta^C = \frac{1}{2}$.

[9] In the online supplementary material, we show that in the collusive contract that maximizes the retailers' profit subject to constraints (3) and (4), retailers would not want to deviate from the collusive contract in a way that maintains collusion. Intuitively, when retailers earn the highest collusive profits possible given that the supplier agrees to participate in collusion (i.e., (4) binds), retailers cannot offer a more profitable contract that maintains collusion.

5.1 Competitive static equilibrium with homogeneous retailers

We start with the static competitive equilibrium that the three firms play in case collusion breaks down. At the beginning of each period, both retailers offer the contract $\left(T^C, w^C\right)$ that the supplier accepts. Then, in stage 2, both retailers set p^C and equally split the market. Each retailer earns $(p^C - w^C)\frac{Q(p^C)}{2} - T^C$ and the supplier earns $\pi_S^C \equiv w^C Q(p^C) + 2T^C$.

In any such equilibrium $p^C = w^C$, because in the second stage retailers play the Bertrand equilibrium given w^C. Therefore, there is no competitive equilibrium with $T^C > 0$. There is no competitive equilibrium with $T^C < 0$ either: The supplier can profitably deviate from such an equilibrium by accepting only R_i's offer. To see why, note that R_i expects that in equilibrium the supplier accepts both of the retailers' offers. Accordingly, in stage 2 R_i sets the equilibrium price p^C. The supplier's profit from accepting only R_i's offer is $w^C Q(w^C) + T^C$ – higher than the profit from accepting both offers, $w^C Q(w^C) + 2T^C$ whenever $T^C < 0$. Therefore, in all competitive equilibria, $T^C = 0$.

Next, consider a potential contract deviation. When w^C is too low, R_i may make a contract deviation with $w_i > w^C$. Such a deviation might be profitable for R_i if it motivates the supplier to accept only R_i's offer rather than rejecting R_i's offer and accepting only R_j's offer.[10] The upside for the supplier in accepting R_i's deviation is the higher wholesale price $w_i > w^C$, while the downside is double marginalization: When the supplier accepts only R_i's deviating offer, the supplier expects R_i to set the monopoly retail price corresponding to a wholesale price of w_i, and this involves double marginalization. On the other hand, accepting only R_j's offer induces R_j to set $p_j = w^C$, with no double marginalization. Hence the supplier will accept R_i's contract deviation for a sufficiently low w^C. More specifically, let $p(w_i)$ denote the price that maximizes R_i's monopoly profit, $(p - w_i)Q(p)$. The following lemma characterizes the set of competitive static equilibria:

Lemma 5. *Suppose that retailers are homogeneous and $\delta = 0$. Then, there are multiple equilibria with the contracts $(T^C, w^C) = (0, w^C)$, $w^C \in [w_L, p_M]$, where w_L is the lowest*

[10] If R_i believes that the supplier accepts R_j's offer regardless of w_i, then any $w^C \in [0, p_M]$ and therefore any $\pi_S^C \in [0, p_M Q_M]$ can be an equilibrium. Such beliefs are consistent with the definition of "passive beliefs" in McAfee and Schwartz (1994), in the sense that R_i believes that the supplier behaves as in the equilibrium strategy in its dealings with R_j despite R_i's deviation from equilibrium.

w^C that satisfies:

$$w^C Q(w^C) \geq \max_{w_i}\{w_i Q(p(w_i))\}, \tag{6}$$

and $0 < w_L \leq p_M$. In equilibrium, retailers set p^C and earn 0 and the supplier earns $\pi_S^C \equiv w^C Q(w^C)$, $\pi_S^C \in [w_L Q(w_L), p_M Q_M]$.

Notice that when retailers are fully homogeneous, the monopoly outcome can be an equilibrium. To see why, suppose that R_i expects that R_j will set $w^C = p_M$. The supplier can earn the monopoly profits by accepting R_j's offer exclusively. This is because R_j, who is unaware that he is the exclusive retailer, sets $p^C = w^C = p_M$ and therefore the supplier earns $w^C Q(w^C) = p_M Q_M$ and fully internalizes R_j's profits. This in turn implies that R_i cannot benefit from offering any wholesale price other than $w^C = p_M$.

Yet when retailers have all of the bargaining power, the monopoly outcome is not a very natural equilibrium. It's existence is an artifact of the lack of any degree of differentiation and the fact that vertical contracts are secret. In particular, recall that section 3 showed that with even slight differentiation, retailers gain a positive profit margin that the supplier does not internalize, which provides the supplier with an opportunistic incentive to agree to a secret discount offer by R_i. Accordingly, all pure strategy equilibria with $w^C > 0$ (and consequently, with $T^C \neq 0$) are eliminated.

In what follows, we assume that if vertical collusion breaks down, retailers play a static equilibrium with $\pi_S^C < p_M Q_M$. Intuitively, we show below that a low π_S^C inflicts a harsh punishment on the supplier when deviating from collusion. Retailers have an incentive to coordinate on a static equilibrium that inflicts such a punishment because they want to induce the supplier not to deviate from the collusive scheme . In particular, it would be counterproductive for the retailers, on the punishment path, to offer the supplier the monopoly wholesale price. Even though this is one of the equilibria of the static game, it cannot deter the supplier from deviating from collusion, so for $\delta < \frac{1}{2}$, retailers would then make zero profits. We note that our results below are not qualitatively affected by π_S^C, as long as retailers can coordinate on some $\pi_S^C < p_M Q_M$.

5.2 Conditions for the collusive equilibrium with homogeneous retailers

In this section we solve for a collusive equilibrium in which retailers offer (w^*, T^*) and then set p_M. Each retailer earns $\pi_R(w^*, T^*) = (p_M - w^*)\frac{Q_M}{2} - T^*$ and the supplier earns $\pi_S(w^*, T^*) = w^* Q_M + 2T^*$. As we show below, there can be multiple collusive contracts, which differ in the way profits are divided between retailers and the supplier. As retailers have all of the bargaining power, we focus on the collusive equilibrium that maximizes the retailers' profits, subject to the supplier earning at least the profits in the competitive static equilibrium.

In the case of homogeneous retailers, conditions (3) and (4) become:

$$(p_M - w)\frac{Q_M}{2} - T + \frac{\delta}{1-\delta}\left((p_M - w)\frac{Q_M}{2} - T\right) \geq (p_M - w) Q_M - T, \tag{7}$$

and:

$$\frac{wQ_M + 2T}{1-\delta} \geq wQ_M + T + \frac{\delta}{1-\delta}\pi_S^C. \tag{8}$$

Using the same notation as in the differentiated retailer case, given w, (7) places a lower threshold on T (that is, T should be negative enough), denoted $T_R(w, \delta)$. This is while (8) places an upper threshold on T (that is, T should not be too negative), denoted $T_S(w, \delta)$. Hence, (7) and (8) imply that for vertical collusion to be sustainable, given that the collusive contract was offered and accepted, and given w and δ, it is required that $T_S(w, \delta) \leq T \leq T_R(w, \delta)$. In the proof of proposition 2 below we show that consistent with lemmas 2 and 3, any collusive contract that satisfies $T_S(w, \delta) \leq T \leq T_R(w, \delta)$ requires that $T < 0$ and $w > 0$.

The remaining requirement is that in stage 1, R_i does not find it profitable to deviate to any other contract, $(w_i, T_i) \neq (w^*, T^*)$. As in the differentiated retailers case, we assume that both R_i and the supplier understand whether the deviation will cause collusion to stop or not.

Suppose that R_i and the supplier believe the contract deviation will cause collusion to stop.[11] In the period of any such deviation, R_i can earn at most $p_M Q_M - (w^* Q_M + T^*)$.

[11] As in the differentiated retailer case, it can easily be shown that if they believe that collusion continues, despite the contract deviation, then the contract deviation is not profitable to R_i.

This is because R_i needs to compensate the supplier for his alternative profit from rejecting the deviation, accepting R_j's equilibrium contract and earning $w^* Q_M + T^*$.[12] The following lemma shows that whenever condition (7) holds, such a deviation is not profitable for R_i.

Lemma 6. *Suppose that the contract (w^*, T^*) satisfies (7). Then, under rational beliefs, R_i cannot profitably deviate to any contract offer $(w_i, T_i) \neq (w^*, T^*)$ that stops collusion.*

As in the case with differentiated retailers, the joint, forward-looking supplier takes part in the collusive scheme, and if R_i attempts to deviate from the collusive contract, R_i needs to compensate the supplier for his losses, since otherwise the supplier rejects R_i's offer and refuses to supply the product to R_i.

We can conclude that the collusive contract that maximizes the retailers' profits, (w^*, T^*) solves:

$$\max_{(w,T)} \left\{ (p_M - w) \frac{Q_M}{2} - T \right\},$$

s.t. $T_S(w, \delta) \leq T \leq T_R(w, \delta)$ and $\pi_S(w, T) \geq \pi_S^C$. Proposition 2 characterizes the unique vertical collusive contract that maximizes retailers' profits under the above-mentioned trigger strategy:

Proposition 2. *Suppose that $\delta > 0$. Then, under rational beliefs, in the homogeneous retailer case, there is a unique vertical collusive equilibrium that maximizes the retailers' profits. In this equilibrium:*

$$w^* = \begin{cases} p_M - \dfrac{2\delta^2 \left(p_M Q_M - \pi_S^C\right)}{(1-\delta) Q_M}; & \text{if } \delta \in (0, \frac{1}{2}]; \\ \dfrac{\pi_S^C}{Q_M}; & \text{if } \delta \in \left[\frac{1}{2}, 1\right], \end{cases} \tag{9}$$

and:

$$T^* = \begin{cases} -\dfrac{\delta}{1-\delta}(1-2\delta)(p_M Q_M - \pi_S^C); & \text{if } \delta \in (0, \frac{1}{2}]; \\ 0; & \text{if } \delta \in \left[\frac{1}{2}, 1\right], \end{cases} \tag{10}$$

Proposition 2 yields that in equilibrium the retailers and the supplier earn $\pi_R^* \equiv$

[12] We show in the online supplementary material that rational beliefs following such a contract deviation (if accepted by the supplier) yields a mixed-strategy equilibrium.

$\pi_R(w^*, T^*)$ and $\pi_S^* \equiv \pi_S(w^*, T^*)$ where:

$$\pi_R^* = \begin{cases} \delta\left(p_M Q_M - \pi_S^C\right); & \delta \in (0, \frac{1}{2}]; \\ \frac{1}{2}\left(p_M Q_M - \pi_S^C\right); & \delta \in \left[\frac{1}{2}, 1\right]; \end{cases} \quad \pi_S^* = \begin{cases} (1-2\delta)\, p_M Q_M + 2\delta\pi_S^C; & \delta \in (0, \frac{1}{2}]; \\ \pi_S^C; & \delta \in \left[\frac{1}{2}, 1\right]. \end{cases}$$

5.3 The features of the vertical collusion equilibrium with homogeneous retailers

Let $SA^* = -T^*$ denote the equilibrium slotting allowance. The following corollary describes the features of the vertical collusion equilibrium in the homogeneous retailer case, while figure 2 illustrates the vertical collusion equilibrium as a function of δ.

Corollary 1. *In the vertical collusion equilibrium in the homogeneous case:*

1. *For $\delta \in (0, \frac{1}{2}]$ retailers' one-period profits are increasing with δ while the supplier's one-period profit is decreasing with δ; the equilibrium wholesale price is decreasing with δ; The supplier pays retailers slotting allowances: $SA^* > 0$. The slotting allowances are an inverse U-shaped function of δ.*

2. *For $\delta \in [\frac{1}{2}, 1]$ the equilibrium wholesale price and the firms' profits are independent of δ and retailers do not charge slotting allowances: $SA^* = 0$; the supplier earns its reservation profit (from the competitive equilibrium) and retailers earn the remaining monopoly profits.*

[Figure 2 here]

Figure 2 and part (*1*) of corollary 1 reveal that at $\delta = \frac{1}{2}$, $w^* = \frac{\pi_S^C}{Q_M}$, $SA^* = 0$ and retailers earn most of the monopoly profits. As δ decreases, w^* increases, and retailers gain a lower proportion of the monopoly profits. The equilibrium slotting allowances are an inverse U-shaped function of δ. Finally, at $\delta \to 0$, $w^* \to p_M$, and the supplier earns all of the monopoly profits.

The intuition for these results is as follows. When $\delta = \frac{1}{2}$, retailers are indifferent between colluding with the supplier's participation or not, so no slotting allowances are needed. When δ decreases slightly below $\frac{1}{2}$, retailers rely on the supplier in the following way. First, retailers set $SA^* > 0$ in order to satisfy condition (7) (slotting allowances are needed to deter retailers from deviating). Second, retailers need to motivate the supplier to agree to pay them $SA^* > 0$ in every period, by raising w^*. Third, the increase in w^* has positive repercussions on the retailers' incentive to collude, back through condition (7).

Corollary 1 further characterizes the case of homogeneous retailers for even lower levels of δ. As δ further decreases, both retailers and the supplier have less of an incentive to collude and the parties raise both slotting allowances and wholesale prices to enable collusion. Moreover, the shorter sighted retailers become more dependent on the supplier, who also becomes more short-sighted, which requires retailers to leave the supplier with an increasingly higher share of the collusive profit. At some point, the high slotting allowances hinder the supplier's willingness to participate in the collusive scheme, so for even lower levels of δ, slotting allowances decline, and even higher wholesale prices are used to enable collusion. Since any increase in w^* only further deters retailers from deviating, the parties can always find an SA^* and w^* that satisfy the retailers' and supplier's incentive constraints, so vertical collusion is sustainable for all $\delta > 0$.

As $\delta \to 0$, collusion is still sustainable, with slotting allowances close to zero, and wholesale prices close to the monopoly retail price. As noted in section 5.1, in the homogeneous retailer case, a wholesale price equal to the monopoly price is also one of the multiple equilibria of the static game, absent collusion. Vertical collusion on the monopoly outcome converges to this equilibrium as $\delta \to 0$.

Part 2 of Corollary 1 reveals that when $\delta > \frac{1}{2}$, retailers can maintain horizontal collusion without the supplier's participation. Accordingly, retailers offer the supplier contracts that grant him his profit when collusion breaks down, π_S^C, and do not charge slotting allowances.

We conclude this section by highlighting the policing role of the supplier. The supplier's ability to stop collusion by rejecting one of the retailer's offers forces retailers to share the profits from collusion with the supplier and induces them to punish the supplier for deviations by offering him a low wholesale price along the punishment path. Retailers cannot do better by using a softer trigger strategy, which removes the bite from the supplier's ability to stop vertical collusion by rejecting a retailer's offer. To see why, suppose that whenever R_i observes that R_j didn't carry the product, R_i continues with the collusive equilibrium. R_i stops offering the collusive contract only if R_j carried the product in the previous period but charged a price different than p_M. Under such a trigger strategy, the supplier's decision whether to accept a retailer's offer no longer affects future collusion. The following lemma shows that retailers cannot implement the collusive scheme with this trigger strategy:

Lemma 7. *Suppose that $\delta < \frac{1}{2}$ and retailers do not stop collusion if they observe that one of them did not carry the supplier's product. Then, there are no contracts (w^*, T^*) that can maintain a collusive equilibrium.*

6 Competition among suppliers, exclusive dealing and renegotiation

Until now, we have assumed that the supplier is a monopoly. Because the monopolistic supplier cares about future profits, and because he often cannot achieve the monopoly outcome in a static game, he enables vertical collusion even for $\delta < \frac{1}{2}$, where ordinary horizontal collusion between the retailers breaks down. The monopoly supplier result should carry over to the case of competing suppliers that are highly differentiated. Our results suggest that if a supplier's brand is strong enough so that the supplier makes a positive profit from refusing a retailer's offer and selling only to the competing retailer, then the parties can engage in vertical collusion. The question arises, however, can vertical collusion be used to reach a monopoly outcome with intense competition among suppliers?

This section extends the homogeneous retailer model to the case of multiple homogeneous suppliers. Its main conclusion is that vertical collusion can achieve monopoly pricing in such cases when the collusive equilibrium involves one of the suppliers being offered short-term exclusive dealing agreements by both retailers. The repeated game induces both retailers to keep offering the same supplier to buy exclusively from him, provided that the supplier can inform one retailer (in the form of cheap talk) that the other retailer did not offer the supplier an exclusive dealing agreement and provided that following such transfer of information, the retailer can renegotiate his offer to the supplier. Otherwise, the vertical collusive equilibrium breaks down. We shall first discuss the case without exclusive dealing. Next we will analyze exclusive dealing without renegotiation, and then examine exclusive dealing with renegotiation.

Suppose that the market includes several identical suppliers S_1, $S_2 ... S_n$. S_1 discounts future profits by δ. In the first stage of every period, retailers simultaneously make secret offers to one or more suppliers. Each supplier cannot observe R_1 and R_2's offers to other suppliers, and each retailer cannot observe the competing retailer's offers. All suppliers simultaneously decide whether to accept or reject the contract offers. Then, in the second stage of every period, the homogeneous retailers set prices and decide from which supplier/s to place their input orders.

We ask whether the two retailers can sustain a collusive equilibrium in which they offer only S_1 the contract (w^*, T^*) that S_1 accepts, and then charge consumers p_M. As before, we assume that any observable deviation in period t triggers the competitive equilibrium from period $t+1$ onwards.

Consider the competitive equilibrium in which all firms earn zero. That is, $\pi_S^C = 0$. Unlike the case of a monopolistic supplier, with competing suppliers $w^C = T^C = 0$ is an equilibrium, when R_j expects that R_i offers a contract to some of the competitive suppliers with $w_i = T_i = 0$.

6.1 No exclusive dealing

Suppose first that retailers cannot commit to deal exclusively with S_1. In order to maintain a collusive equilibrium, the collusive contract has to satisfy conditions (7) and (8). The case of interest is $\delta < \frac{1}{2}$, since otherwise retailers can collude without the aid of any supplier. The collusive contract needs to eliminate R_i's incentive to deviate from collusion by offering the collusive contract to S_1 and at the same time making a secret offer to a competing supplier with $w_i = T_i = 0$. To see the profitability of such a deviation, suppose that R_j plays according to the proposed equilibrium by offering (w^*, T^*) to S_1 only, but the deviating retailer, R_i, offers S_1 the collusive contract (w^*, T^*) and at the same time makes a secret offer to one or more of the competing suppliers with $w_i = T_i = 0$. S_1 will accept both retailers' offers, because he is unaware of R_i's secret offer to the competing suppliers. Hence R_i will earn a slotting allowance, $-T^* > 0$, from S_1. Moreover, R_i can then charge consumers a price slightly below p_M, dominate the market and earn $p_M Q_M - T^*$. Therefore, the equilibrium requires that R_i's discounted future profits from the collusive equilibrium, $((p_M - w^*)\frac{Q_M}{2} - T^*)/(1-\delta)$, are higher than a one-period deviation in which R_i buys from a competing supplier, $p_M Q_M - T^*$. However,

$$p_M Q_M - T^* > \frac{\delta p_M Q_M}{(1-\delta)} \geq \frac{(p_M - w^*)\frac{Q_M}{2} - T^*}{1-\delta}, \tag{11}$$

where the first inequality follows because $T^* < 0$ and $\delta < \frac{1}{2}$, and the second inequality follows from the definition of π_R^*. This implies that R_i will deviate from this collusive equilibrium by making the secret offer to the competing supplier.[13] The following corollary summarizes this result:

Corollary 2. *Suppose that the upstream market includes multiple homogeneous suppliers S_1,S_2,..., S_n. Then, if retailers cannot offer one of the suppliers, with a discount factor δ, an exclusive dealing contract, and $\delta < \frac{1}{2}$, vertical collusion is not sustainable.*

6.2 Exclusive dealing, communication and renegotiation

Consider first exclusive dealing without communication and renegotiation. Suppose that in every period, each retailer can offer a contract (w_i, T_i, ED) where ED denotes an exclusive dealing clause according to which in the current period, the retailer cannot make contract offers to competing suppliers. The exclusive dealing clause is valid for one period only. We maintain our assumption that contracts are secret and therefore a retailer cannot observe whether the competing retailer offered S_1 an exclusive dealing clause.

We first show that retailers' ability to commit to an exclusive dealing clause, by itself, is not enough to support a collusive equilibrium when $\delta < \frac{1}{2}$. Consider a proposed collusive equilibrium in which in every period, the two retailers offer S_1 a contract (w^*, T^*, ED), where w^* and T^* are the same as in proposition 1 S_1 accepts both offers and then retailers set p_M. This cannot be an equilibrium, because R_i will find it optimal not to make an offer to S_1 and instead offer a deviating contract $w_i = T_i = 0$ to one or more of the competing suppliers. Applying "rational beliefs" to this deviation cannot yield pure strategies: If S_1 – observing that R_i didn't make him an offer – believes that R_i obtained the product from a different supplier and will undercut the monopoly price,

[13]Notice that this argument holds for any (w^*, T^*) that satisfy conditions (7) and (8), and not just for the collusive contract that maximizes the retailers' profit.

then S_1 will not accept R_j's equilibrium offer. But if R_i believes that S_1 will reject R_j's equilibrium offer, R_i will monopolize the retail market even if he does not undercut the collusive retail price, so he would not undercut it. However, there is a mixed-strategy equilibrium following such a deviation, in which S_1 accepts R_j's offer with a very small probability and R_i mixes between setting $p_M - \varepsilon$ and p_M. This equilibrium is consistent with rational beliefs, and R_i's expected profit is $p_M Q_M - \varepsilon$, while the dominant supplier's expected profit is 0.[14] From (11), whenever $\delta < \frac{1}{2}$, R_i prefers making this one-period deviation and earning 0 in all future periods to maintaining collusion.

The reason why collusion breaks down even when retailers can commit to an exclusive dealing contract is that we did not allow S_1 to inform R_j that R_i did not offer S_1 an exclusive dealing contract. Suppose now that S_1 can engage in bilateral communication, followed by renegotiation, with each retailer: In the first stage of every period, after retailers made their contract offers but before S_1 accepted them, S_1 can inform R_j that R_i deviated from the collusive contract and that S_1 didn't accept his offer. This communication is non-verifiable, and consists of "cheap talk" that S_1 can convey to retailers, regardless of whether it is true or not. R_j can then withdraw his original offer and make an alternative offer to S_1.[15] The supplier then accepts or rejects the alternative offer and the game moves to the second stage as in our base model.[16] Suppose that R_j interprets any such communication as a signal that R_i deviated from the collusive contract. Accordingly, R_j finds it optimal to replace the original offer with the contract $w_j = T_j = 0$. As we show below, such a belief is rational, because S_1 reports to R_j that R_i deviated from the collusive contract if and only if R_i indeed deviated.

To show that now the collusive contract defined in proposition 1, alongside an exclusive dealing clause, can maintain collusion, we can follow the same steps as in our base model. First, condition (7) is still necessary, because it ensures that given that both retailers deal exclusively with S_1 and offered the collusive contract in the first stage, R_i will not undercut the monopoly price in the second stage.

Next consider S_1's incentive constraint. Suppose that both retailers offered S_1 the

[14] For a detailed description of this mixed-strategy equilibrium see the online supplementary material.

[15] It can be shown that the results remain the same when R_i cannot remove the original offer and instead can make a second offer such that S_1 chooses between the two offers.

[16] It can be shown that such communication and renegotiation has no effect on the collusive equilibrium when there is a monopoly supplier. The intuition is that if the monopoly supplier rejects R_j's offer, the supplier will never want to inform R_i that R_i is a monopoly retailer.

equilibrium contract. If S_1 rejects one of the offers, say, R_i's offer, he has no incentive to report it to R_j, because then R_j will offer a contract $w_j = T_j = 0$ and S_1 will earn 0. This means that S_1's profit from accepting only one of the equilibrium offers is $w^* Q_M + T^*$, as in our base model, and S_1's incentive constraint is identical to (8) (after substituting $\pi_S^C = 0$). This further implies that S_1 has no incentive to report a deviation to R_j when there was no deviation.[17]

Finally, consider the possibility that R_i deviates in the first period by offering a contract different than (w^*, T^*, ED). If the deviating contract includes an exclusive dealing clause and differs only because $(w_i, T_i) \neq (w^*, T^*)$, the same reasoning as in section 5 (lemma 6) holds. If the deviating contract does not include an exclusive dealing clause, then regardless of w_i and T_i, S_1 will rationally believe that R_i offered a competing supplier a contract $w_i = T_i = 0$ and plans to cut the monopoly price. Given this belief, there is no point in accepting R_j's equilibrium contract and paying him a slotting allowance, and instead S_1 will inform R_j of the deviation. R_j in turn will offer S_1 a contract $w_j = T_j = 0$ that S_1 will accept. This makes R_i's deviation unprofitable to begin with. The following corollary summarizes this result:

Corollary 3. *Suppose that the upstream market includes homogeneous suppliers S_1,... ,S_n. Then, if retailers can offer S_1 an exclusive dealing contract, communication and renegotiation between S_1 and retailers is possible, and $\delta < \frac{1}{2}$, there is a collusive equilibrium in which in every period the two retailers sign an exclusive dealing contract (w^*, T^*, ED) with S_1, where (w^*, T^*) are as defined in proposition 2.*

Notice that the exclusive dealing contract with the joint supplier facilitates vertical collusion even though the commitment to buy exclusively from the supplier is of a short-term, i.e., retailers commit to the supplier for only one period. In every period, R_i is induced by the repeated game to offer S_1 exclusivity, because R_i knows that if he does not, S_1 will inform R_j of this and R_j will undercut the collusive price and monopolize the market.

[17] Notice that if S_1 accepts R_i's equilibrium offer and falsely reports to R_j that R_i deviated from collusion, S_1 will earn $T^* < 0$.

7 Policy Implications

Our results have several policy implications.

First, they shed a new light on short-term exclusive dealing agreements in which buyers agree to buy from a single supplier. As shown in section 6.2, the ability of retailers to promise one of the suppliers to buy only from him, even for a single period, facilitates vertical collusion and enables monopoly retail prices. Current antitrust rules in the US and in the EU, however, see such short-term exclusive dealing agreements as automatically legal. For example, the US Court of Appeals in Roland Machinery Company v. Dresser Industries, Inc.,[18] ruled that "[e]xclusive-dealing contracts terminable in less than a year are presumptively lawful ... ". Similarly, in Methodist Health Services Corporation v. OSF Healthcare System,[19] the dominant hospital in a certain region, facing competition from only one other hospital, entered exclusive dealing agreements with the local insurance companies. The District Court dismissed the antitrust claim because the exclusive dealing contracts were short term agreements. The court stresses that "[e]ven an exclusive-dealing contract covering a dominant share of a relevant market need have no adverse consequences if the contract is let out for frequent rebidding."[20] Even though the dominant hospital kept winning these bids, the court approved of the exclusive dealing commitments, because in each bid the other hospital had the opportunity to compete.[21] Our results, however, imply that in such scenarios, one of the suppliers may keep winning these bids for the wrong reasons: not because he offers lower prices or better terms, but rather because he can better enforce a multi-period tacit collusion scheme (e.g., because this supplier is forward looking and the competing supplier isn't). That is, in our framework, exclusive dealing becomes self-enforcing: the collusive equilibrium repeatedly induces both retailers to offer to buy exclusively from the same supplier. The European Commission too (EC Commission (2009)) says, in its guidelines, that "[i]f

[18](7th Cir.) 749 F.2d 380, 395 (1984).

[19](Central District of Illinois, Peoria Division) 2016 U.S. Dist. LEXIS 136478.

[20]Id. at 150.

[21]Id. at 149. For other cases where a US court declined to intervene against exclusivity by a dominant supplier due to its short term see Louisa Coca-Cola Bottling Co. v. Pepsi-Cola Metropolitan Bottling Co., Inc. (US District Court for The Eastern District of Kentucky, Ashland Division) 94 F. Supp. 2d 804 (1999). See also Insulate SB, Inc. v. Advanced Finishing Systems, Inc., (Court of Appeals 8th Cir.), 797 F.3d 538 (2015), denying the claim of a buyer of insulation material supplied by a dominant supplier (Graco Minnesotta Inc) through distributors. According to this claim, Graco and its distributors were engaged in a conspiracy designed to have the distributors buy the product exclusively from Graco, so as to enable the distributors to raise the price they charged up to supra-competitive levels.

competitors can compete on equal terms for each individual customer's entire demand, exclusive purchasing obligations are generally unlikely to hamper effective competition unless the switching of supplier by customers is rendered difficult due to the duration of the exclusive purchasing obligation."[22] This approach too overlooks the anticompetitive effect of short-term exclusive dealing obligations exposed by our results. This anticompetitive effect hinges neither on the duration of the exclusive dealing obligation nor on competing suppliers' ability to compete for each retailer's entire demand. Notice that even though suppliers 2, 3, ... etc. in our model offer both retailers a perfect substitute that can fulfill all of their demand, in the collusive equilibrium we identify, retailers are nevertheless induced to offer supplier 1 exclusivity over and over again.

Facts that make short-term exclusive dealing particularly prone to the anticompetitive effects we identify are: The same supplier keeps winning the retailers' business (as opposed to cases where different suppliers win the exclusive contracts each time or where each retailer offers exclusivity to a different supplier); The exclusive supplier pays retailers fixed fees (as Lemma 2 shows, vertical collusion requires slotting allowances.)

The second policy implication involves transfer of information between a supplier and his customers. Antitrust law generally allows a supplier to reveal to one customer what another customer had offered him.[23] As shown in section 6.2, however, if supplier 1 can reveal to one retailer that the competing retailer had not offered supplier 1 an exclusive dealing contract, vertical collusion is enabled. Had such transfer of information been under the threat of antitrust liability, the vertical collusive scheme would have been more likely to break down. The general justification for allowing exchange of information between a supplier and a retailer regarding dealings of the supplier with the competing retailer is that the supplier is supposedly trying to improve the deal, using competition among buyers over his product. Note, however, that the competitive threat we identify does not really stem from information the dominant supplier reveals to one retailer regarding a better deal offered by the competing retailer. On the contrary, the particular type of information transfer we are discussing concerns the supplier revealing to one retailer that the other retailer actually offered him a worse deal: one without exclusive dealing. Hence, the justification for a soft antitrust approach does not hold in

[22] See also Case C-234/89 Stergios Delimitis Henninger Brau AG, [1991] ECR 935.
[23] Compare supra note 1.

this case.

The third policy implication concerns the antitrust treatment of a supplier's refusal to deal with a retailer. The supplier's ability to unilaterally reject a deviating retailer's contract offer plays a key role in the sustainability of vertical collusion. By contrast, under US antitrust law, a supplier's refusal to deal with a retailer due to the retailer's vigorous competition with other retailers is often deemed automatically legal. The famous "Colgate doctrine",[24] cited in recent cases as well, "protects a manufacturer who communicates a policy and then terminates distribution agreements with those who violate that policy ... and a distributor is free to acquiesce in the manufacturer's demand in order to avoid termination."[25] Such behavior, if not accompanied by additional evidence of an anticompetitive agreement between the supplier and retailers, is generally considered unilateral action, invoking no antitrust claim.[26] Hence, our results suggest that antitrust courts and agencies, in appropriate cases, should be more strict toward such unilateral refusals by a dominant supplier. In particular, if evidence of the anticompetitive reasons for such refusal is presented, an illegal agreement between the supplier and retailers should be more easily inferred. Furthermore, if the evidence suggests that a dominant supplier's unilateral refusal to deal with a retailer stems from the retailer's attempt to deviate from tacit collusion, antitrust courts and agencies should be able to condemn such a refusal as illegal monopolization. When market conditions are prone to vertical collusion, had such a retailer possessed an antitrust claim against the dominant supplier for such refusal, vertical collusion would be more likely to break down. By contrast, US antitrust law is commonly understood not to include such a prohibition.[27]

Finally, our results imply that slotting allowances may be more anticompetitive than the current economic literature predicts. According to the economic literature to date, one retailer needs to observe its rival's contract with the supplier in order for slotting allowances to facilitate downstream collusion. By contrast, this paper shows that slotting allowances might be anti-competitive even in the common case when contracts between

[24] See United States v. Colgate & Co., 250 U.S. 300 (1919).

[25] See State of New York, v. Tempur-Pedic International, Inc., (Supreme Court of New York, 30 Misc. 3d 986 (2011).

[26] See Monsanto Co. v. Spray-Rite Service Corp., 465 U.S. 752 (1984), Costco Wholesale Corporation v. Johnson & Johnson Vision Care, Inc., (United States District Court for The Middle District of Florida, Jacksonville Division), 2015 U.S. Dist. Lexis 168581; Kaplow (2016) (criticizing the case law's attempt to distinguish between unilateral and concerted behavior).

[27] See Areeda and Hovenkamp (2015).

suppliers and retailers are secret. Usually, a retailer cannot observe its rivals' contracts with the supplier. As noted, exchange of information among competing retailers regarding their commercial terms with a common supplier would most probably be condemned as an antitrust violation.[28] We show that even though vertical contracts are secret, retailers know that the supplier observes both contracts and has an incentive to maintain vertical collusion. Without slotting allowances, vertical collusion breaks down. In some cases, slotting allowances are paid by suppliers as "compensation" for intense competition among retailers over selling the supplier's brand.[29] Our results imply that such scenarios deserve softer antitrust treatment, provided that the claim of compensation for intense competition is not a sham. In our framework, during or after a price war between retailers, when collusion collapses, slotting allowances are no longer used. On the contrary, when vertical collusion collapses, the supplier stops paying retailers slotting allowances in our model, in order to implicitly punish retailers for not adhering to the collusive scheme.

8 Conclusion

We examine the features of collusion in a repeated game involving retailers and their joint supplier. Our model of vertical collusion has two main features. First, all three firms equally care about the future and they all participate in the collusive scheme. Second, vertical contracts are secret: a retailer cannot observe the bilateral contracting between the competing retailer and the supplier. Retailers gain from vertical collusion, because it enables them to charge the monopoly retail price even for discount factors that would not have enabled ordinary horizontal collusion among them, and they receive slotting allowances from the supplier as an implicit prize for participating in the collusive scheme. The supplier gains from vertical collusion, because he collects a higher wholesale price and makes a higher profit than absent vertical collusion. This occurs even when retailers have all the bargaining power, and even when retailers are differentiated, where the supplier's difficulty in receiving a high wholesale price is at its peak. Also, it occurs despite the fact retailers are too impatient to sustain horizontal collusion, and despite

[28]See sources cited supra note 1.
[29]See, e.g., Moulds (2015).

the fact the supplier is as impatient as retailers are.

This result could naturally carry over to multiple suppliers, as long as differentiation among them is strong enough. With intense competition among homogeneous suppliers, vertical collusion is sustained by short-term exclusive dealing commitments by retailers with one of the suppliers. Exclusive dealing can enable vertical collusion, however, only when the supplier is allowed to tell one retailer (in the form of cheap talk) that the competing retailer did not offer an exclusive dealing contract. Vertical collusion enables the exclusive supplier to raise the wholesale price he charges, while enabling retailers to charge monopoly retail prices, despite the potential for intense competition among suppliers and among retailers.

An interesting question is whether a third party other than the supplier could replace the supplier's role in facilitating collusion. In our framework, the supplier is the most natural candidate for such a task for three reasons. First, recall that in the collusive scheme, the payment from the retailer to the third-party is contingent on the retailer's actual sales (through the wholesale price). Second, and more importantly, it is essential that the third party not only detect contract deviations, but also block the deviating retailer from receiving access to the product. Third, using the supplier rather than a third party can help avoid antitrust scrutiny. For future research, it would be interesting to examine whether retailers can use a third party who lacks the abilities described above to assist their collusive scheme.

Our results have various policy implications: antitrust courts and agencies should reconsider their automatic approval of short-term exclusive dealing agreements; transfer of information from a supplier to his buyer regarding whether the competing buyer offered to buy exclusively from the supplier may raise antitrust concerns; a dominant supplier's refusal to deal with a retailer on account of the retailer engaging in downstream competition deserves more antitrust attention; and slotting allowances can facilitate collusion even when vertical contracts are secret.

Appendix A

Below are the proofs of lemmas 1 – 7 and propositions 1 and 2.

Proof of Lemma 1:

Consider a potential equilibrium in which retailers set the monopoly price. In such an equilibrium, if it were to exist, retailers need to offer the supplier $w^C = \hat{w}(p_M)$, where $\hat{w}(p_M) > 0$ is the solution to $p(\hat{w}, p_M) = p_M$. However, suppose now that R_i decides to deviate to $(w_i, T_i) \neq (w^C, T^C)$ that triggers the joint beliefs by R_i and the supplier that the supplier will accept the deviation as well as R_j's offer and R_i will set $p(w_i; p^C)$. The supplier accepts the deviation if:

$$w_i q(p(w_i; p^C), p^C) + w^C q(p^C, p(w_i; p^C)) + T^C + T_i \geq w^C \widehat{q}(p^C) + T^C,$$

where the left hand side is the supplier's profit from accepting both offers and the right hand side is the supplier's profit from rejecting R_i's offer and accepting only R_j's offer. Solving for T_i, R_i earns from this deviation:

$$\begin{aligned} &(p(w_i; p^C) - w_i) q(p(w_i; p^C), p^C) - T_i = \\ &\left[p(w_i; p^C) q(p(w_i; p^C), p^C) + w^C q(p^C, p(w_i; p^C))\right] - w^C \widehat{q}(p_M). \end{aligned} \tag{12}$$

Equation (12) shows that as is standard in the literature on vertical relations with secret contracts, R_i sets w_i so as to maximize his and the supplier's joint profit, ignoring R_j's profit.[30] The first term in the squared brackets of (12) is the joint profit of R_i and the supplier from selling product i. The second term is the supplier's own revenue from selling product j. When retailers are perfect substitutes, then $w^C = \hat{w}(p^C) = p^C$. In such a case $w_i = w^C$ maximizes (12) and there is an equilibrium in which retailers implement the monopoly outcome by charging $w^C = p_M$. For the same reason, there are also equilibria with $w^C < p_M$. Yet, when retailers are differentiated, $p(w^C; p^C) > w^C$ (equivalently, $w^C = \hat{w}(p^C) < p^C$). In this case, R_i does not fully internalize the effect of w_i on the total profit from selling product j. Since $q(p^C, p(w_i; p^C))$ is increasing with w_i

[30] Equation (12) is equivalent to equation (1) in O'Brien and Shaffer (1992).

but $w^C < p^C$, R_i sets $w_i < w^C$, and the monopoly outcome cannot be an equilibrium.[31]

■

Proof of Lemma 2:

Solving (3) for T, we have:

$$T_R(w,\delta) \equiv -\frac{1-\delta}{\delta}\left[(p(w;p_M)-w)q(p(w;p_M),p_M)+\frac{\delta}{1-\delta}\pi_R^C-\frac{(p_M-w)q_M}{(1-\delta)}\right],$$

We will now show that $T_R(0,\delta^C)=0$ and $T_R(0,\delta)$ is increasing with δ. Evaluating at $w=0$:

$$T_R(0,\delta) = -\frac{1-\delta}{\delta}\left[p(0;p_M)q(p(0;p_M),p_M)+\frac{\delta}{1-\delta}\pi_R^C-\frac{p_M q_M}{(1-\delta)}\right].$$

The term inside the squared brackets in $T_R(0,\delta)$ is identical to condition (1) when it holds in equality and therefore equals 0 at $\delta=\delta^C$. Hence the effect of δ on $T_R(0,\delta)$ is:

$$\frac{\partial T_R(0,\delta)}{\partial\delta} = \frac{p(0;p_M)q(p(0;p_M),p_M)-p_M q_M}{\delta^2} > 0,$$

where the inequality follows because $p(0;p_M)$ maximizes $pq(p,p_M)$. ■

Proof of Lemma 3:

Solving (4) for T, we have:

$$T_S(w,\delta) \equiv -w\left[\frac{2q_M}{1-\delta}-\widehat{q}(p_M)\right]\frac{1-\delta}{1+\delta}.$$

The first part of lemma 3 follows directly from the definition of $T_S(w,\delta)$. The effect of δ on $T_S(w,\delta)$ is:

$$\frac{\partial T_S(w,\delta)}{\partial\delta} = -2w\frac{\widehat{q}(p_M)-q_M}{(1+\delta)^2} < 0,$$

where the inequality follows because $\widehat{q}(p_M) > q_M$.

[31] This argument also holds when R_i expects that whenever the supplier accepts the deviation, the supplier does not accept R_j's offer. This is because in such a case R_i would like to set $w_i = 0$ to eliminate the double marginalization problem.

This implies that the supplier will not participate in the collusive scheme when $T < 0$ if $w = 0$. Recall that the supplier's incentive constraint requires that $T > T_S(w, \delta)$, which cannot hold when $T_S(0, \delta) = 0$ while $T < 0$. ■

Proof of Proposition 1:

We start by showing that condition (5) is necessary for collusion when $\delta < \delta^C$. Recall that given the collusive contract, collusion is sustainable only if $T_R(w, \delta) > T_S(w, \delta)$. Accordingly, it suffices to show that if condition (5) fails, then $T_R(w, \delta) < T_S(w, \delta)$. Recall from lemmas 2 and 3 that evaluated at $w = 0$, $T_R(0, \delta^C) = T_S(0, \delta^C)$, and the gap $T_R(0, \delta) - T_S(0, \delta)$ is increasing in δ. This implies that for $w = 0$ and $\delta < \delta^C$, $T_R(0, \delta) < T_S(0, \delta)$ and collusion is impossible. Moreover, if condition (5) fails, such that when evaluated at $\delta = \delta^C$ and $w = 0$, $T_R(w, \delta) - T_S(w, \delta)$ is decreasing in w, then $T_R(w, \delta) < T_S(w, \delta)$ for all $\delta < \delta^C$ and $w \geq 0$ (and not just for w close to zero). This is because $T_R(w, \delta) - T_S(w, \delta)$ is concave in w for all δ and w: the second derivative of $T_R(w, \delta) - T_S(w, \delta)$ with respect to w is:

$$\frac{\partial^2 (T_R(w, \delta) - T_S(w, \delta))}{\partial^2 w} = \frac{1 - \delta}{\delta} \frac{\partial q(p(w; p_M), p_M)}{\partial p_i} \frac{\partial p(w; p_M)}{\partial w} < 0,$$

where the inequality follows (for all δ and w) because $q(p_i, p_j)$ is decreasing in p_i and $p(w; p_M)$ is increasing in w.

Next, we move to showing that when condition (5) holds and δ is slightly below δ^C, it is possible to find w^* such that $T_R(w, \delta) > T_S(w, \delta)$ when $w > w^*$. Since $T_R(w, \delta)$ and $T_S(w, \delta)$ are continuous in δ, a marginal decrease in δ below δ^C results in a marginal decrease in $T_R(0, \delta) - T_S(0, \delta)$ below 0. Yet, when (5) is strictly positive, then there is a cutoff in w, w^*, such that $T_R(w, \delta) > T_S(w, \delta)$ when $w > w^*$, where w^* is the solution to $T_R(w^*, \delta) = T_S(w^*, \delta)$. Finally, we have that $w^* \to 0^+$ as $\delta \to \delta^{C-}$ because $T_S(0, \delta) = 0$, and $T_R(0, \delta) \to 0^-$ as $\delta \to \delta^{C-}$. ■

Proof of Lemma 4:

Suppose that R_i offered a contract $(w_i, T_i) \neq (w^*, T^*)$ such that both the supplier and R_i understand that this deviation (if accepted) stops collusion. That is, R_i plans to set the price $p(w_i; p_M)$. The supplier accepts the deviation if:

$$w_i q(p(w_i; p_M), p_M) + w^* q(p_M, p(w_i; p_M)) + T^* + T_i \geq w^* \widehat{q}(p^M) + T^*,$$

where the left hand side is the supplier's profit from accepting both R_j's collusive contract and R_i's deviating contract and the right hand side is the supplier's profit from accepting R_j's collusive contract and rejecting R_i's deviating contract. Solving for T_i, R_i earns from this deviation

$$(p(w_i; p_M) - w_i) q(p(w_i; p_M), p_M) - T_i =$$

$$[p(w_i; p_M) q(p(w_i; p_M), p_M) + w^* q(p_M, p(w_i; p_M))] - w^* \widehat{q}(p_M).$$

Let $\tilde{w}(w^*)$ denote the w_i that maximizes R_i's profit from deviating from collusion. There is no loss of generality in looking at the w_i that maximizes R_i's profit, because any other w_i that triggers the beliefs that R_i plans to deviate from collusion makes the deviation less profitable. R_i will not deviate if:

$$\frac{(p_M - w^*) q_M - T^*}{(1-\delta)} \geq \tag{13}$$

$$[p(\tilde{w}(w^*); p_M) q(p(\tilde{w}(w^*); p_M), p_M) + w^* q(p_M, p(\tilde{w}(w^*); p_M))] - w^* \widehat{q}(p_M) + \frac{\delta}{1-\delta} \pi_R^C.$$

Let $T_{RS}(w, \delta)$ denote the solution to (13) in equality. We have that (13) holds if $T^* < T_{RS}(w^*, \delta)$, where:

$$T_{RS}(w^*, \delta) \equiv \delta \left[-\pi_R^C - \frac{(1-\delta) p(\tilde{w}(w^*); p_M) q(p(\tilde{w}(w^*); p_M), p_M) - (p_M - w^*) q_M}{\delta} \right]$$

$$+ w^* (\widehat{q}(p_M) - q(p_M, p(\tilde{w}(w^*); p_M))(1-\delta).$$

Recall that condition (3) requires that $T^* < T_R(w^*, \delta)$. Consequently, when $T_{RS}(w^*, \delta) > T_R(w^*, \delta)$, condition (13) is not binding on the optimal contract. Let us compare $T_{RS}(w^*, \delta)$ with $T_R(w^*, \delta)$:

Evaluating $T_{RS}(w^*, \delta)$ at $w^* = 0$, the second term in $T_{RS}(w^*, \delta)$ vanishes. The term inside the squared brackets becomes identical to $T_R(0, \delta)$. To see why, notice that $\tilde{w}(0) = 0$, because when $w^* = 0$, $\tilde{w}(0) = 0$ maximizes $p(w; p_M) q(p(w; p_M), p_M)$. Hence,

$T_{RS}(0,\delta) = \delta T_R(0,\delta)$. Since lemma 2 establishes that $T_R(0,\delta) < 0$ for $\delta < \delta^C$, and since $\delta < 1$, we have that $T_{RS}(0,\delta) > T_R(0,\delta)$. Since $T_{RS}(w^*,\delta)$ and $T_R(w^*,\delta)$ are continuous in w^*, we have that $T_{RS}(w^*,\delta) > T_R(w^*,\delta)$ as long as w^* is not too high.

Notice that we consider the case where retailers are sufficiently differentiated such that if the supplier accepts R_i's contract deviation, the supplier also accepts R_j's equilibrium contract. When retailers are close substitutes, the supplier may choose to accept only one of the offers. As shown in the proof of lemma 6, the results above hold also in the special case where retailers are perfect substitutes, in that condition (7) prevents R_i from making a contract deviation.■

Proof of Lemma 5:

We will proceed in two steps. In the first step, we will show that if (6) does not hold then R_i finds it optimal to deviate to a contract that motivates the supplier to reject R_j's offer (and that this deviation is impossible if (6) holds). In the second step we show that R_i cannot profitably deviate to a contract that motivates the supplier to accept R_j's offer.

We first show that if (6) does not hold, R_i can make a profitable deviation. Since $p(w) > w$ and $pQ(p)$ is concave in p:

$$\max_{w^C}\{w^C Q(w^C)\} = p_M Q_M > \max_{w_i}\{w_i Q(p(w_i))\} \geq w^C Q(w^C)\big|_{w^C=0},$$

implying that there is a w_L such that (1) holds for $w_C \in [w_L, p_M]$ and does not hold otherwise, where $w_L > 0$. Suppose that (6) does not hold. Then R_i can deviate to (T_i, w_i) such that $w_i Q(p(w_i)) > w^C Q(w^C)$. If the supplier accepts the contract, it is rational (for both the supplier and R_i) to expect that the supplier does not accept R_j's offer and that R_i sets $p(w_i)$. Given these expectations, the supplier agrees to the deviating contract if $w_i Q(p(w_i)) + T_i \geq w^C Q(w^C)$, or $T_i = w^C Q(w^C) - w_i Q(p(w_i))$. R_i earns from this deviation:

$$(p(w_i) - w_i)Q(p(w_i)) - T_i = p(w_i)Q(p(w_i)) - w^C Q(w^C) > w_i Q(p(w_i)) - w^C Q(w^C) > 0,$$

where the first inequality follows because $p(w_i) > w_i$ and the second inequality follows because whenever (6) does not hold it is possible to find w_i such that $w_i Q(p(w_i)) > w^C Q(w^C)$. Since in equilibrium R_i earns 0, R_i finds it optimal to deviate. Now suppose that (6) holds. Then, there is no w_i that ensures that the supplier does not accept R_j's offer.

Next we turn to the second step, of showing that R_i cannot make a profitable deviation when R_i anticipates that the supplier accepts R_j's equilibrium offer. Suppose that R_i deviates to $(T_i, w_i) \neq (0, w^C)$ so that if the supplier accepts the deviation, the supplier continues to play the equilibrium strategy of accepting R_j's offer, $(0, w^C)$. R_i therefore expects that R_j will be active in the market and will set $p^C = w^C$. The deviation can be profitable to R_i only if $w_i < w^C$, so that in stage 2 R_i can charge a price slightly lower than w^C and dominate the market. To convince the supplier to accept the deviating contract, R_i sets T_i so that the supplier is indifferent between accepting both offers and accepting just R_j's equilibrium offer: $w_i Q(w^C) + T_i \geq w^C Q(w^C)$, or $T_i \geq (w^C - w_i) Q(w^C)$. But then R_i earns at most $(w^C - w_i) Q(w^C) - T_i \leq 0$. We therefore have that R_i cannot offer a profitable deviation from the equilibrium $(0, w^C)$ if R_i believes that the supplier accepts R_j's equilibrium offer. ■

Proof of Lemma 6:

As shown in the online supplementary material, the highest expected profit that R_i can make in such a deviation is $p_M Q_M - \varepsilon - (w^* Q_M + T^*)$. Letting $\varepsilon \to 0$, R_i does not deviate iff:

$$\frac{(p_M - w^*)\frac{Q_M}{2} - T^*}{1-\delta} \geq p_M Q_M - (w^* Q_M + T^*),$$

which is equivalent to condition (7). ■

Proof of Proposition 2:

We start by proving that as in the differentiated retailer case, when $\delta < \delta^C = \frac{1}{2}$, any contract that satisfies $T_S(w, \delta) \leq T \leq T_R(w, \delta)$ must involve $T < 0$ and $w > 0$.

Lemma 8. *If $\delta < \frac{1}{2}$, then any collusive equilibrium has to involve negative fees, $T^* < 0$.*

Proof of lemma 8:

Recall that $T_R(w, \delta)$ is the highest T that satisfies the retailers' incentive constraint defined by equation (7). Solving (7) for T, yields that condition (7) holds iff $T < T_R(w, \delta)$, where:

$$T_R(w,\delta) = (p_M - w)\frac{Q_M(2\delta - 1)}{2\delta}.$$

When $\delta < \frac{1}{2}$ and $p_M > w$, we have that $T_R(w,\delta) < 0$. If however $p_M < w$, then from the retailer's profit function: $\pi_R(w,T) = (p_M - w)\frac{Q_M}{2} - T > 0$, it follows that when $p_M < w$, it has to be that $T < 0$. ■

Lemma 9. *When $T^* < 0$, condition (8) requires that $w^* > \frac{\pi_S^C}{Q_M}$ and $\delta > 0$. As T^* decreases, a higher w^* is needed to maintain condition (8).*

Proof of lemma 9:

Recall that $T_S(w,\delta)$ is the lowest T that satisfies the supplier's incentive constraint defined by equation (8). Solving (8) for T^* yields that (8) holds iff $T > T_S(w,\delta)$, where:

$$T_S(w,\delta) \equiv -\frac{\delta(wQ_M - \pi_S^C)}{1+\delta}. \tag{14}$$

Since from lemma 8, $T^* < 0$, the condition $0 > T \geq T_S(w,\delta)$ requires that $T_S(w,\delta) < 0$, which holds iff $w > \frac{\pi_S^C}{Q_M} > 0$ and $\delta > 0$. Moreover, as T^* decreases, a higher w^* is needed to maintain condition (8) because $T_S(w,\delta)$ is decreasing with w^*. ■

Next, we turn to solve for the set of (w^*, T^*) that satisfy (7), (8) (or $T_R(w,\delta) > T_S(w,\delta)$) and $\pi_S(w,T) \geq \pi_S^C$. First, we have that $T_R(w,\delta) > T_S(w,\delta)$ if:

$$w > p_M - \frac{2\delta^2\left(p_M Q_M - \pi_S^C\right)}{(1-\delta)\,Q_M}. \tag{15}$$

Substituting $T = T_S(w,\delta)$ into $\pi_S(w,T)$ we have:

$$\pi_S\left(w, T_S(w,\delta)\right) = \frac{1-\delta}{1+\delta}wQ_M + \frac{2\delta}{1+\delta}\pi_S^C > \pi_S^C \Leftrightarrow w > \frac{\pi_S^C}{Q_M}. \tag{16}$$

Comparing the right-hand-sides of (15) and (16), the former is higher than the latter iff $\delta < \frac{1}{2}$. We conclude that (7), (8) and $\pi_S(w,T) \geq \pi_S^C$ hold for any $T^* = T_S(w^*,\delta)$ and w^*, where:

$$w^* \geq w^E \equiv \begin{cases} p_M - \frac{2\delta^2\left(p_M Q_M - \pi_S^C\right)}{(1-\delta)Q_M}; & \text{if } \delta \in (0, \frac{1}{2}]; \\ \frac{\pi_S^C}{Q_M}; & \text{if } \delta \in \left[\frac{1}{2}, 1\right]. \end{cases} \tag{17}$$

Next, we solve for the w^* that maximizes $\pi_R(w, T_S(w,\delta))$, where:

$$\pi_R\left(w, T_S(w,\delta)\right) = \left(p_M - w\right)\frac{Q_M}{2} - \frac{\delta\left(\pi_S^C - wQ_M\right)}{(1+\delta)}.$$

Differentiating $\pi_R(w, T_S(w,\delta))$ with respect to w yields:

$$\frac{\partial \pi_R(w, T_S(w,\delta))}{\partial w} = -\frac{(1-\delta)\,Q_M}{2(1+\delta)} < 0.$$

Therefore, the most profitable collusive equilibrium involves $w^* = w^E$ which yields (9). Substituting $w^* = w^E$ into $T_S(w^*,\delta))$ yields (10). ■

Proof of Lemma 7:

Condition (7) is still necessary to support a collusive equilibrium. Turning to the supplier's incentive constraint, given that both retailers offer the equilibrium collusive contracts, the supplier's decision on whether to accept both of them or just one is not going to affect the future. Hence the supplier's incentive constraint could be written as:

$$w^*Q_M + 2T^* = w^*Q_M + T^*,$$

where the left-hand-side is the supplier's profit from accepting the two equilibrium contracts and the right-hand-side is the supplier's profit from accepting only one of them. This condition requires that $T^* = 0$. However, condition (7) cannot hold if $T^* \geq 0$ and $\delta < \frac{1}{2}$, implying that this alternative trigger strategy cannot maintain a collusive equilibrium. ■

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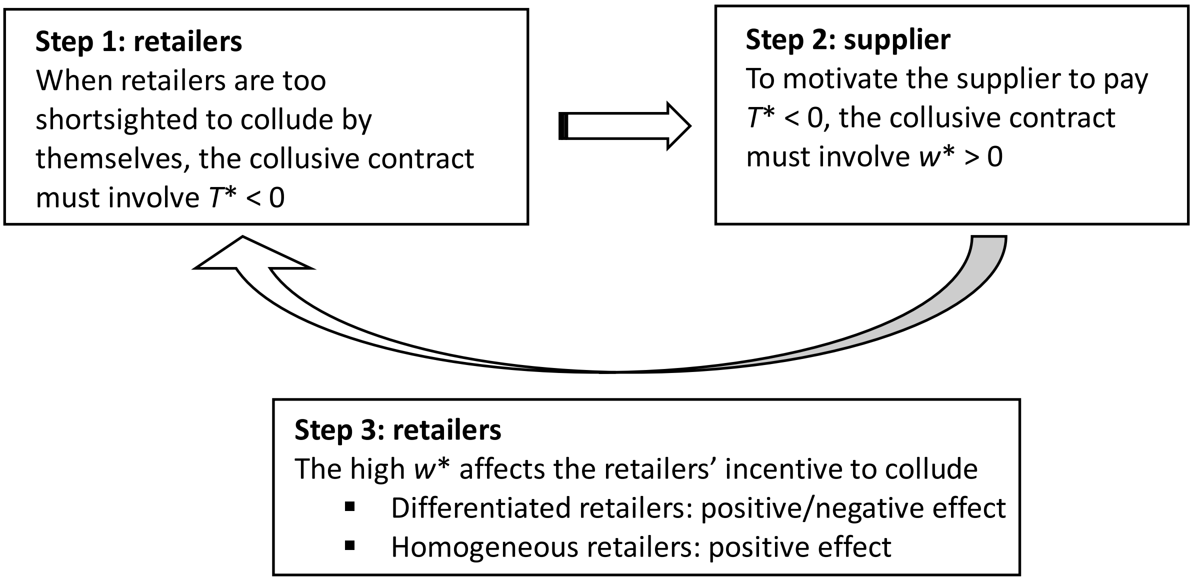
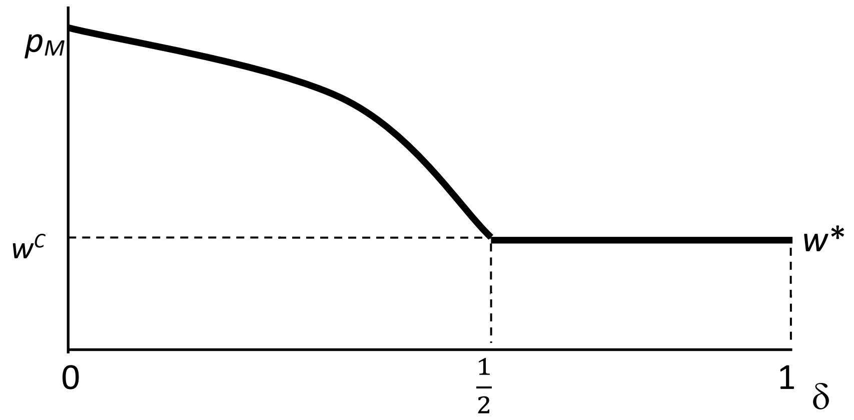
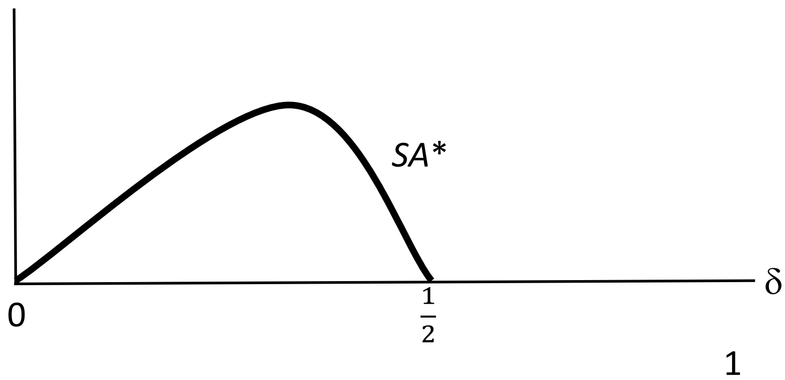


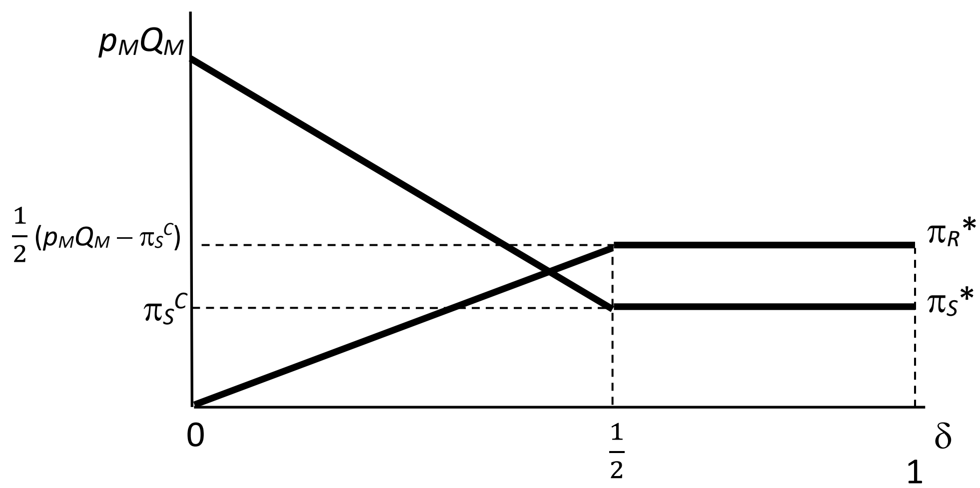
Figure 1: the mechanism that enables vertical collusion for $\delta < \delta^c$



Panel (a): The equilibrium *w as a function of δ**



Panel (b): The equilibrium *SA as a function of δ**



Panel (c): The firms' equilibrium profits as a function of δ

(when $(p_M Q_M - \pi_S{}^C)/2 > \pi_S{}^C$)

Figure 2: The features of the vertical collusion equilibrium as a function of δ