# Vertical Collusion to Exclude Product <br> Improvement* 

By David Gilo ${ }^{\dagger}$ and Yaron Yehezkel ${ }^{\ddagger}$


#### Abstract

A manufacturer of an established product repeatedly interacts with a retailer that can sell an inferior new product thereby improving it. The manufacturer's exclusionary strategy consists of a permanently below-cost wholesale price and "vertical collusion" with the retailer to exclude via a future reward of a reduced fixed fee. The latter tool is available only in an infinite game. Although contracts include fixed fees, the retailer sells the new product more than what maximizes industry profits. Exclusive dealing or a vertical merger between the manufacturer of the established product and the retailer replicate the vertically integrated outcome and increase prices.


Keywords: dynamic vertical relations, vertical collusion, predatory pricing, exclusive dealing, vertical mergers

JEL Classification Numbers: L41, L42, K21,D8

[^0]
## I. Introduction

It is often the case that an established product is challenged by a new product, which is first inferior to the established product. Nevertheless, this new product may often have the potential for improvement, if it is supplied and sold to consumers. For example, consumers may not be aware of the new product's virtues, and only if it is sold, and consumers try it, their demand for it increases (Guadagni and Little [1983]; Seetharaman et al [1999]; Osborne [2011]). This is particularly the case in online markets, where consumers can rate new products and online retailers can publish how many consumers had bought them, both of which can enhance consumers' awareness to the product and its perceived quality (Danaher et al [2003]). ${ }^{1}$ Consumer's demand for the new product could also be initially low due to their costs of switching or adaptation costs, and the retailer too could face temporary costs of adapting to it. Another prominent case is that of learning by doing: when the new product is produced in high enough quantities, its cost of production may decline and/or its quality may improve (e.g., Glocka et al [2019]; Löbberding and Madlener [2019]). The new product could also be initially inferior due to network effects: Demand for it could increase when it has been consumed by a critical mass of users.

An important feature of these sorts of improvement of the new product is that they often depend on the retailer, who needs to sacrifice its short term profits by holding the initially inferior new product so as to improve it. The economic literature on exclusion of new products has dealt with the allegation, made at the time by the Chicago School of economics, that a new product will be excluded only if it is inferior (e.g., Bork, [1993]). Some of the post-Chicago economic literature examines whether superior new products would nevertheless be excluded (e.g., the exclusive dealing literature we cite in the literature review), and another part of this literature studies whether exclusion of inferior products could nevertheless harm welfare (e.g., Edlin [2002] and [2018]). Our focus, instead, is on the prevalent case where the new product is initially inferior, but, if sold by the retailer, it improves. In our framework, therefore, the retailer faces a tradeoff and needs to sacrifice short-term profits to improve the new product. In this paper, we ask what is the best strategy for an incumbent firm to compete against a new product which is not consistently inferior nor is it

[^1]consistently superior to the incumbent firm's product: Instead, the new product is initially inferior but becomes superior when sold by a retailer.

When answering this research question, we explore the exclusionary strategies employed by the manufacturer of the established product when banned by antitrust courts from imposing explicit exclusive dealing or merging with the retailer. This reveals new exclusionary strategies that take advantage of the retailer's above-mentioned tradeoff between its short-term sacrifice and long-term gain from improving the new product. This, in turn, provides a new theory of harm for explicit exclusive dealing and vertical mergers, because these practices raise prices in our framework. At the same time, since we find that a vertically separated industry without exclusive dealing may overaccommodate the new product from a social perspective, exclusive dealing and vertical mergers in our framework can at times be pro-competitive, by preventing such over-accommodation.

To capture these insights, we study dynamic vertical relations between a manufacturer of an established product, selling to an independent (monopoly) retailer. ${ }^{2}$ In each period, the retailer can buy the established product, but it can also sell a competing product that is currently inferior, and can improve in the next period if sold at some threshold quantity. When the new product improves, it becomes more profitable than the established product. The manufacturer and the retailer interact for an infinite number of periods and have the same discount factor. When explicit exclusive dealing is banned by antitrust rules (i.e., the terms of the contract between the manufacturer and the retailer may not explicitly rely on whether the retailer sells the new product), each period the manufacturer of the established product offers the retailer a contract involving a two-part tariff. Then, the retailer decides whether to sell the established product, the new one, or both. We then explore the possibility that the manufacturer of the established product is allowed to impose exclusive dealing, or vertically merge with the retailer. In addition, we study the implications of a vertical merger between the manufacturer of the new product and the retailer.

We find that when explicit exclusive dealing is banned by antitrust rules, the manufacturer's exclusionary strategy depends on how patient the retailer is. For low discount factors, the man-

[^2]ufacturer excludes the new product with a simple two-part tariff and maximizes industry profits with monopoly pricing. For intermediate discount factors, the manufacturer's exclusionary strategy consists of indefinitely charging a below-cost wholesale price and a positive fixed fee. We refer to this exclusionary tool as "persistent predatory pricing", in order to distinguish it from standard predatory pricing (where the manufacturer lowers its price in the present in order to raise prices and recoup its losses in the future). Unlike standard predatory pricing, in our framework the manufacturer charges a below-cost wholesale price indefinitely and recoups its losses immediately via the fixed fee. ${ }^{3}$ Intuitively, for these relatively low discount factors, it is more profitable to exclude the new product by inflating the retailer's short-term sacrifice from improving the new product. Since we assume explicit exclusive dealing is banned by antitrust law, the fixed fee cannot affect this short-term sacrifice. The only way the manufacturer can intensify the short-term sacrifice is via a below-cost wholesale price. Yet, this form of 'persistent predatory pricing' reduces the retail price below the monopoly price and sacrifices industry profits. For high discount factors, the manufacturer mitigates (but does not eliminate) this sacrifice of industry profits by also reducing the fixed fee. This reduction diminishes the retailer's long-term benefit from improving the new product: if the new product is improved, the retailer no longer buys the established product, so it sacrifices the future benefit of the reduced fixed fee. We call this second exclusionary tool 'vertical collusion to exclude', because as in horizontal collusion between competing firms, this tool crucially relies on intertemporal reciprocity between two firms, here a manufacturer and a retailer. In particular, this tool is only possible in an infinitely repeated game, as with standard collusion. The manufacturer exploits the horizon being infinite in order to induce the retailer to exclude the new product via implicit future rewards: If the retailer avoids the new product in the current period, the manufacturer implicitly rewards the retailer via a reduced fixed fee in future periods. When vertical collusion to exclude is used, the wholesale price is less predatory, and the retail price is closer to the monopoly price. Like horizontal collusion among competitors, vertical collusion to exclude harms competition, by excluding a new and improvable product, yet the ongoing 'collusion'

[^3]is not among competitors, but between the manufacturer and the retailer, and the outcome of such collusion is exclusion rather than price fixing. Unlike the other exclusionary tool, of 'persistent predatory pricing', vertical collusion to exclude relies on the retailer's expectation for a future prize for excluding the new product.

We show that this second tool, of vertical collusion to exclude, is not available when the horizon is finite, even when the number of periods converges to infinity. In the finite-horizon case, since there is a last period in which the new product cannot be improved, the only exclusionary strategy is persistent predatory pricing. The reason is that the parties foresee this last period, in which their relationship ends, which completely eliminates a future promise of a reduced fixed fee as an exclusionary strategy. Hence for both exclusionary tools to be available, the parties must interact indefinitely so as to develop mutual trust that their relationship has no ultimate period.

These unique exclusionary strategies, which hinge solely on the retailer's tradeoff between its short-term sacrifice versus its long-term benefit of improving the new product, have important implications. First, because the parties fail to maximize industry profits, they will over-accommodate the new product compared to what maximizes industry profits, and for some parameters, also compared to what maximizes social welfare. Intuitively, because the manufacturer offers a two-part tariff contract, it might be expected that the manufacturer will offer a contract that implements the joint profit-maximizing outcome and then use the fixed fee to share this profit with the retailer. Yet, because of the dynamic nature of the game (the new product being improvable by the retailer), our model exposes a divergence between the vertically integrated outcome and vertical separation, a divergence that does not occur if the new product is not improvable by the retailer. In the infinite horizon case, when the second exclusionary strategy, of vertical collusion to exclude, is available, this strategy helps mitigate the divergence between the vertically separated and vertically integrated results. Accordingly, with an infinite horizon, there is more exclusion of the new product than in a finite game in which the number of periods approaches infinity, and the retail price is higher.

To avoid the divergence between the vertically integrated and vertically separated outcomes, the manufacturer needs to impose explicit exclusive dealing, where contractual terms explicitly change if the retailer buys the new product. The parties can similarly avoid this divergence via a vertical merger between the manufacturer of the established product and the retailer. With such practices,
the parties accommodate the new product as would a vertically integrated industry, and there is no persistent predatory pricing or vertical collusion to exclude. Absent our framework of the retailer being able to improve the new product, the manufacturer does not need exclusive dealing or a vertical merger to achieve the vertically integrated outcome, as predicted by the Chicago School.

This result, in turn, presents important policy implications. The first are with regard to the antitrust assessment of explicit exclusive dealing (broadly interpreted as any case where contractual terms depend on whether the retailer sells the new product). When such explicit vertical restraints are banned, all the manufacturer can do is set a two part tariff, in which, as we show, persistent predatory pricing emerges, lowering the retail price below monopoly levels. Hence, if persistent predatory pricing is allowed by antitrust law (as is indeed the case under U.S. antitrust law), then explicit exclusive dealing (even for a short period) or a vertical merger, raise the retail price to the monopoly price. This presents a new theory of harm for exclusive dealing and vertical mergers, as they will increase the retail price. On the other hand, absent exclusive dealing or a vertical merger, we show that the new product might be over-accommodated from a social perspective. Hence our results present a new setting in which these practices may be pro-competitive.

Furthermore, because vertical collusion to exclude, with its future promise of a reduced fixed fee, hinges on an infinite horizon, in any finite game the wholesale price is more predatory than in the infinite game. This implies that in dynamic, technologically changing industries, where the horizon of the vertical relationship is finite because products become obsolete and new products emerge, ${ }^{4}$ vertical mergers and explicit exclusive dealing raise prices more (and therefore may deserve stricter antitrust treatment), than in other industries, in which the horizon is infinite.

Finally, our results imply that persistent predatory pricing, in our dynamic framework, may be socially beneficial, because it helps alleviate monopoly pricing. Persistent predatory pricing has ambiguous welfare effects when it comes to over or under-accommodation of the new product. Depending on the circumstances of each case, absent persistent predatory pricing, the new product might be over-accommodated from a social perspective, while with such predatory pricing, it might be over-excluded.

[^4]Our paper contributes to the literature on vertical foreclosure via pricing in dynamic settings, such as Fumagalli and Motta [2013], Cabral and Riordan [1994], Segal and Whinston [2007], Fudenberg and Tirole [2000] and Carlton and Waldman [2002]'s discussion of virtual tying. In all of these papers consumers buy only one of the products each period. Hence there is no scope for the role that persistent predatory pricing plays in our model.

In Asker and Bar-Isaac [2014], the supplier persuades retailers not to accommodate the entrant by repeatedly sharing its monopoly profits with them. We contribute to this paper by considering the unique exclusionary practices that arise when the entrant is initially inferior to the incumbent, but improves if sold by the retailer. This is why, for example, persistent predatory pricing arises in our model and does not arise in Asker and Bar-Isaac [2014].

Edlin [2018 and 2002] shows that above-cost predatory pricing may harm competition and raise prices since a less efficient entrant keeps out of the market due to the incumbent's threat to lower prices upon entry, and thus such lower prices do not occur. In our framework, a persistent predatory wholesale price occurs in equilibrium, so exclusion is accompanied by more competitive pricing.

Our results hinge on the manufacturer's unique exclusionary strategies when explicit exclusive dealing (i.e., making the contract depend on whether the buyer buys the new product) is banned by antitrust law. Hence our paper differs from the literature discussing other motivations for explicit exclusive dealing, tying, loyalty discounts, or refusals to deal, either in dynamic settings, such as Fumagalli and Motta [2020]; Calzolari and Spagnolo [2020]; Carlton and Waldman [2002; 2012], or static settings, such as Aghion and Bolton [1987]; Whinston [1990]; Fumagalli and Motta [2006]; Fumagalli et al [2012]; Chao et al [2018]; Rasmusen, Ramseyer, and Wiley [1991]; Segal and Whinston [2000]; Bernheim and Whinston [1998]; Spector [2011]; and Simpson and Wickelgren [2007]. None of these papers studies the case where the buyer sacrifices short-term profits to improve the new product. By studying the manufacturer's unique exclusionary strategies emerging in this case when explicit exclusive dealing is banned, our analysis provides a new theory of harm (the price-increasing effect), along-side a new claim for pro-competitive effects (prevention of over-accommodation), for explicit exclusive dealing.

Another strand of the literature our paper contributes to is on vertical collusion that achieves horizontal price fixing (as in Piccolo and Miklós-Thal [2012] and Gilo and Yehezkel [2020]). Their
collusion-facilitating strategies involve an above-cost wholesale price and negative fixed fees, while our exclusionary strategy involves a below-cost wholesale price and positive fixed fees.

The next section describes the model. We solve for the equilibrium when exclusive dealing is banned in Section III. Section IV considers the cases where exclusive dealing is allowed by antitrust rules, a ban on below cost wholesale prices and vertical mergers. Section V analyzes the welfare and the policy implications of these practices. Section VI discusses extensions to the base model and Section VII concludes.

## II. The model

Consider an upstream manufacturer that supplies product 1 to a monopolistic retailer. ${ }^{5} \mathrm{~A}$ second product, denoted product 2 , is also available to the retailer. Product 2 can be either a new entrant into the market or an existing rival. The marginal costs of producing product $i$ is $c_{i}$, where $c_{2}>c_{1}$. Without loss of generality, we assume that product 2 is sold by a perfectly competitive fringe and is available to the retailer at marginal cost. ${ }^{6}$ The manufacturer and retailer play a repeated game and discount future profits by $\delta(0 \leq \delta \leq 1)$.

In the first period, consumers view products 1 and 2 as perfect substitutes. The inverse demand function facing the retailer is $p(q)$, where the price $p$ and marginal revenue are decreasing with total quantity $q$. Let $q_{i}^{V I}$ denote the quantity that maximizes the one-period vertically integrated (monopoly) profits when selling product $i,\left(p\left(q_{i}\right)-c_{i}\right) q_{i}$, and let $\pi_{i}^{V I} \equiv\left(p\left(q_{i}^{V I}\right)-c_{i}\right) q_{i}^{V I}$ denote the monopoly profits. Because $c_{2}>c_{1}$, the one-period profits of selling product 1 are higher than selling product 2: $q_{1}^{V I}>q_{2}^{V I}$ and $\pi_{1}^{V I}>\pi_{2}^{V I}$. Yet, suppose that stand alone, product 2 provides positive monopoly profits: $\pi_{2}^{V I}>0$.

Product 2 can improve if sold by the retailer. As noted in the introduction, such improvement can take several forms, all of which can be captured by our analysis. In particular, suppose that if the retailer sells at least the quantity $q_{2}=\underline{q}<q_{2}^{V I}$ for one period, in all future periods product 2

[^5]substantially improves, and the monopoly profit from selling product 2 is $\pi_{2 H}^{V I}$, where $\pi_{2 H}^{V I}>\pi_{1}^{V I} .{ }^{7}$ Hence, if product 2 is improved, it is profitable to offer only product 2 instead of product $1 .{ }^{8}$ It is possible to improve product 2 in any period. As long as product 2 is not improved, product 1 remains the superior product.

The two firms play an infinitely repeated game. In each period, the manufacturer offers a two-part-tariff contract $\left(w_{1}, t_{1}\right)$ valid for the current period, where $w_{1}$ is a wholesale price per-unit and $t_{1}$ is a fixed fee. The retailer chooses whether to sell products 1,2 or both and sets the total quantity to consumers. We assume that the manufacturer remains active in the market in all periods.

We first solve for a dynamic vertical integration benchmark. We ask when it is profitable for a vertically integrated firm to carry product 2 in the first period in order to improve it in the following periods. When the integrated firm does not carry product 2 in any period, it earns $\pi_{1}^{V I}$ in each period. When the integrated firm chooses to carry product 2 , it sets $q_{2}=\underline{q}$ and chooses $q_{1}$ to maximize:

$$
\pi_{12}^{V I}\left(q_{1}\right) \equiv p\left(q_{1}+\underline{q}\right)\left(q_{1}+\underline{q}\right)-c_{1} q_{1}-c_{2} \underline{q} .
$$

Let $\widehat{q}_{1}$ denote the quantity that maximizes $\pi_{12}^{V I}\left(q_{1}\right)$ and let $\pi_{12}^{V I} \equiv \pi_{12}^{V I}\left(\widehat{q}_{1}\right)$. It is straightforward to show that $\widehat{q}_{1}=q_{1}^{V I}-\underline{q}$. Intuitively, the total quantity is chosen based on the marginal cost of the last unit, $c_{1}$, because $\underline{q}<q_{2}^{V I}<q_{1}^{V I}$. We note that the crucial feature of $\widehat{q}_{1}$ for our results is that $\widehat{q}_{1}<q_{1}^{V I}$, which holds even if the two products are differentiated. Because $\widehat{q}_{1}<q_{1}^{V I}$, there is a short-term sacrifice in selling product 2 along-side product 1 . When selling both products, the vertically integrated firm earns $\pi_{12}^{V I}<\pi_{1}^{V I}$ in the current period, followed by $\pi_{2 H}^{V I}>\pi_{1}^{V I}$ in all future periods. Hence, it is profitable for a vertically integrated firm to sell product 2 in the first period in order to improve it in the subsequent periods iff:

$$
\begin{equation*}
\pi_{12}^{V I}+\frac{\delta}{1-\delta} \pi_{2 H}^{V I} \geq \frac{\pi_{1}^{V I}}{1-\delta} \tag{1}
\end{equation*}
$$

[^6]Let $\delta^{V I}$ denote the solution to (1) in equality, where $0<\delta^{V I}<1$. Intuitively, it is profitable for the vertically integrated firm to sell both products (despite product 2's inferiority) if it cares more about future profits and is willing to sacrifice current profits to increase future profits (thanks to product 2's improvement).

## III. Vertical-Separation, non-EXClusive contract

In this section we assume that the manufacturer and retailer are vertically separated and the manufacturer is banned by antitrust law from imposing explicit exclusive dealing. Accordingly, the manufacturer cannot change the fixed fee or wholesale price if the retailer sells product 2 .

We start by considering the manufacturer's optimal exclusionary offer: the contract that motivates the retailer to sell only product 1 . Then we move to the manufacturer's optimal accommodating contract, which induces the retailer to sell product 2 alongside product 1. Finally, we compare the two in order to identify when the manufacturer prefers accommodation to exclusion.

## The manufacturer's optimal exclusionary contract

Consider first an equilibrium in which the manufacturer sets $\left(w_{1}, t_{1}\right)$ so as to motivate the retailer not to sell product 2. In this equilibrium, the manufacturer offers $\left(w_{1}, t_{1}\right)$ in every period and the retailer accepts and sells only product $1 .{ }^{9}$ After accepting the manufacturer's exclusionary offer $\left(w_{1}, t_{1}\right)$, the retailer sets the quantity $q_{1}\left(w_{1}\right)$ that maximizes $\left(p\left(q_{1}\right)-w_{1}\right) q_{1}$ and earns $\pi_{1}^{R}\left(w_{1}\right)-t_{1}$, where $\pi_{1}^{R}\left(w_{1}\right) \equiv\left(p\left(q_{1}\left(w_{1}\right)\right)-w_{1}\right) q_{1}\left(w_{1}\right)$. We have that $\pi_{1}^{R}\left(c_{1}\right)=\pi_{1}^{V I}$. The manufacturer earns in every period $\pi_{1}^{M}\left(w_{1}\right)+t_{1}$, where $\pi_{1}^{M}\left(w_{1}\right) \equiv\left(w_{1}-c_{1}\right) q_{1}\left(w_{1}\right)$.

The retailer can deviate from this equilibrium in two ways. First, the retailer can reject the manufacturer's offer completely and sell only product 2 . In this case, the retailer sells the quantity $q_{2}^{V I}$ of product 2, earns $\pi_{2}^{V I}$ in the current period, and then earns $\pi_{2 H}^{V I}$ in all future periods. Alternatively, the retailer may choose to accept the manufacturer's offer of ( $w_{1}, t_{1}$ ), but deviate from the exclusionary equilibrium by selling both products instead of only product 1 . In this case, the

[^7]retailer sells $q_{2}=\underline{q}$ and $q_{1}=\widehat{q}_{1}\left(w_{1}\right)$ where $\widehat{q}_{1}\left(w_{1}\right)$ maximizes:
$$
\pi_{12}^{R}\left(q_{1}, w_{1}\right) \equiv p\left(q_{1}+\underline{q}\right)\left(q_{1}+\underline{q}\right)-w_{1} q_{1}-c_{2} \underline{q} .
$$

We have that $\widehat{q}_{1}\left(w_{1}\right)=q_{1}\left(w_{1}\right)-q$. Hence, $\widehat{q}_{1}\left(w_{1}\right)$ is decreasing with $w_{1}$ and $\widehat{q}_{1}\left(c_{1}\right)=\widehat{q}_{1} .{ }^{10}$ The retailer's current period profit from accepting the manufacturer's two part tariff and also selling product 2 is $\pi_{12}^{R}\left(w_{1}\right) \equiv \pi_{12}^{R}\left(q_{1}\left(w_{1}\right), w_{1}\right)$, where $\pi_{12}^{R}\left(c_{1}\right)=\pi_{12}^{V I}$. Then, in all future periods, the retailer earns $\pi_{2 H}^{V I}$. Therefore, given the manufacturer's offer $\left(w_{1}, t_{1}\right)$, the retailer does not deviate from the equilibrium in which product 2 is excluded if:

$$
\begin{equation*}
\underbrace{\frac{\pi_{1}^{R}\left(w_{1}\right)-t_{1}}{1-\delta}}_{\text {selling 1 }} \geq \max \{\underbrace{\pi_{2}^{V I}+\frac{\delta}{1-\delta} \pi_{2 H}^{V I}}_{\text {selling } 2}, \underbrace{\pi_{12}^{R}\left(w_{1}\right)-t_{1}+\frac{\delta}{1-\delta} \pi_{2 H}^{V I}}_{\text {selling } 1+2}\} . \tag{2}
\end{equation*}
$$

The left-hand side is the retailer's profit from selling only product 1 in all periods. The right hand side contains the retailer's two options of deviating from the exclusionary equilibrium. We call these three options "selling 1 " "selling 2 " and "selling $1+2$ ", as denoted in (2). If the retailer deviates, whether it prefers selling 2 to selling $1+2$ depends on the manufacturer's proposed contract ( $w_{1}, t_{1}$ ). To see this, comparing the two terms in the right-hand-side of (2), selling 2 is binding if:

$$
\begin{equation*}
t_{1}>\underline{T}\left(w_{1}\right) \equiv \pi_{12}^{R}\left(w_{1}\right)-\pi_{2}^{V I} . \tag{3}
\end{equation*}
$$

The fee $\underline{T}\left(w_{1}\right)$ is the difference between the retailer's profits from selling 2 and its profits from selling $1+2$. It boils down to the difference between the retailer's first-period profits from selling 2 and $1+2$, because the profits in the subsequent periods from selling only the improved product 2 are the same in both cases. Intuitively, for selling 2 to bind, the manufacturer's offer has to be oppressive enough so that the retailer would rather sell only inferior product 2 than sell both products. Accordingly, when only selling 2 binds $\left(t_{1}>\underline{T}\left(w_{1}\right)\right.$ ), then solving (2) for $t_{1}$ (while ignoring

[^8]the option of selling $1+2$ ) the manufacturer can set $t_{1}$ as high as:
\[

$$
\begin{equation*}
t_{1} \leq T_{2}\left(w_{1}, \delta\right) \equiv \pi_{1}^{R}\left(w_{1}\right)-\pi_{2}^{V I}-\delta\left(\pi_{2 H}^{V I}-\pi_{2}^{V I}\right) \tag{4}
\end{equation*}
$$

\]

Likewise, when selling $1+2$ binds $\left(t_{1} \leq \underline{T}\left(w_{1}\right)\right)$, then solving (2) for $t_{1}$ (while ignoring the option of selling 2) the manufacturer can set $t_{1}$ as high as:

$$
\begin{equation*}
t_{1}<T_{12}\left(w_{1}, \delta\right) \equiv \frac{1}{\delta}\left[\pi_{1}^{R}\left(w_{1}\right)-\pi_{12}^{R}\left(w_{1}\right)-\delta\left(\pi_{2 H}^{V I}-\pi_{12}^{R}\left(w_{1}\right)\right)\right] . \tag{5}
\end{equation*}
$$

Let $\widetilde{w}_{1}(\delta)$ denote the solution to $\underline{T}\left(w_{1}, \delta\right)=T_{2}\left(w_{1}, \delta\right)=T_{12}\left(w_{1}, \delta\right)$. The following lemma shows that there is a unique $\widetilde{w}_{1}(\delta)$, such that selling $1+2$ (selling 2) binds if $w_{1}$ is above (below) $\widetilde{w}_{1}(\delta)$. The lemma also characterizes the features of $\widetilde{w}_{1}(\delta)$ :

Lemma 1. (When do selling $1+2$ or selling 2 bind under vertical separation?)
(i) When $w_{1}<\widetilde{w}_{1}(\delta), \underline{T}\left(w_{1}, \delta\right)<T_{2}\left(w_{1}, \delta\right)<T_{12}\left(w_{1}, \delta\right)$, selling 2 binds and the manufacturer sets $t_{1}=T_{2}\left(w_{1}, \delta\right)$.
(ii) When $w_{1} \geq \widetilde{w}_{1}(\delta), \underline{T}\left(w_{1}, \delta\right) \geq T_{2}\left(w_{1}, \delta\right) \geq T_{12}\left(w_{1}, \delta\right)$, selling $1+2$ binds and the manufacturer sets $t_{1}=T_{12}\left(w_{1}, \delta\right) ;$
(iii) For $\delta=0, \widetilde{w}_{1}(0)=c_{2}, \widetilde{w}_{1}(\delta)$ is decreasing with $\delta$ and crosses $c_{1}$ at $\widetilde{\delta}$, where $\widetilde{\delta}$ is the solution to $\widetilde{w}_{1}(\widetilde{\delta})=c_{1}$.
( $i v$ ) For $w_{1}=c_{1}$, selling $1+2$ binds under vertical separation for values of $\delta$ under which a vertically integrated firm would sell only product $1: \widetilde{\delta}<\delta^{V I}$.

Parts (i) - (iii) of Lemma 1 establish the conditions in which selling 2 or selling $1+2$ bind, as a function of $w_{1}$. As we show below, part (iv) will be relevant for our result of over accommodation of product 2 from an industry perspective. The intuition for Lemma 1 can be understood by fixing $t_{1}$ while considering the effect of $w_{1}$ or the other way around. Given that the retailer agrees to pay $t_{1}$ and carries product 1 , it has an incentive to sell product 2 as well in order to improve it if $w_{1}$ is higher than the threshold $\widetilde{w}_{1}(\delta)$ because product 2 is relatively inexpensive compared to product 1. This is why selling $1+2$ binds. An alternative way to see the same intuition is that given a high
$w_{1}$, the manufacturer can charge a low $t_{1}$ (as $\pi_{1}^{R}\left(w_{1}\right)$ is low). This in turn reduces the retailer's incentive to sell only 2 instead of only 1 , so again selling $1+2$ binds. The opposite intuition holds if $w_{1}$ is lower than the threshold $\widetilde{w}_{1}(\delta) . \widetilde{w}_{1}(\delta)$ is decreasing in $\delta$, because the more patient is the retailer, the more weight it places on the long-term gain from improving product 2, and the more likely is this long-term gain to outweigh the short-term loss from selling both products.

The next step is to solve for the manufacturer's optimal contract given that it chooses to exclude product 2. The manufacturer earns $\Pi_{1}^{M}\left(w_{1}\right) \equiv \pi_{1}^{M}\left(w_{1}\right)+t_{1}$. Substituting $t_{1}=T_{2}\left(w_{1}, \delta\right)$ when $w_{1}<\widetilde{w}_{1}(\delta)$ and $t_{1}=T_{12}\left(w_{1}, \delta\right)$ when $w_{1} \geq \widetilde{w}_{1}(\delta)$, and denoting $\pi_{1}^{V I}\left(w_{1}\right) \equiv \pi_{1}^{M}\left(w_{1}\right)+\pi_{1}^{R}\left(w_{1}\right)$, the manufacturer's discounted sum of exclusionary profits can be written as:

$$
\frac{\Pi_{1}^{M}\left(w_{1}\right)}{1-\delta}= \begin{cases}\frac{\pi_{1}^{V I}\left(w_{1}\right)-\pi_{2}^{V I}}{1-\delta}-\frac{\delta}{1-\delta}\left(\pi_{2 H}^{V I}-\pi_{2}^{V I}\right) ; & w_{1}<\widetilde{w}_{1}(\delta)  \tag{6}\\ \frac{1}{\delta}\left(\pi_{1}^{R}\left(w_{1}\right)-\pi_{12}^{R}\left(w_{1}\right)\right)-\frac{\pi_{2 H}^{V I}-\pi_{1}^{V I}\left(w_{1}\right)}{1-\delta} ; & w_{1} \geq \widetilde{w}_{1}(\delta)\end{cases}
$$

It is clear from (6) that its first line is maximized at $w_{1}=c_{1}$. Let $w_{12}(\delta)$ denote the $w_{1}$ that maximizes the second line in $(6), w_{1}^{E}(\delta)$ denote the $w_{1}$ that maximizes (6), and $t_{1}^{E}(\delta)$ the corresponding fixed fee. Hence, as shown in Proposition 1 below, the exclusionary contract is:

$$
w_{1}^{E}(\delta)=\left\{\begin{array}{ll}
c_{1} ; & \delta \in[0, \widetilde{\delta}] ;  \tag{7}\\
\widetilde{w}_{1}(\delta) ; & \delta \in[\widetilde{\delta}, \widetilde{\widetilde{\delta}}] ; \\
w_{12}(\delta) ; & \delta \in[\widetilde{\widetilde{\delta}}, 1] ;
\end{array} \quad t_{1}^{E}(\delta)= \begin{cases}T_{2}\left(c_{1} ; \delta\right) & \delta \in[0, \widetilde{\delta}] ; \\
\underline{T}\left(\widetilde{w}_{1}(\delta)\right) ; & \delta \in[\widetilde{\delta}, \widetilde{\widetilde{\delta}}] ; \\
T_{12}\left(w_{12}(\delta), \delta\right) ; & \delta \in[\widetilde{\widetilde{\delta}}, 1] ;\end{cases}\right.
$$

where $\widetilde{\delta}$ is the solution to $\widetilde{w}_{1}(\delta)=c_{1}$ and $\widetilde{\widetilde{\delta}}$ is the solution to $\widetilde{w}_{1}(\delta)=w_{12}(\delta)$.
Proposition 1 below characterizes two tools that the manufacturer uses to exclude product 2 . The first exclusionary tool is persistent predatory pricing, i.e., charging a below-cost wholesale price indefinitely, to inflate the retailer's short-term sacrifice from improving product 2 . The second exclusionary tool is vertical collusion to exclude, i.e., implicitly rewarding the retailer with a future reduction of the fixed fee, to shrink the retailer's long-term gain from improving product 2. The profitability of each exclusionary tool depends on how patient the retailer is.

Proposition 1. (The features of the exclusionary contract as a function of $\delta$ ):
(i) For $\delta \in[0, \widetilde{\delta}]$, the manufacturer uses neither persistent predatory pricing nor vertical collusion to exclude: $w_{1}^{E}(\delta)=c_{1}$ is constant in $\delta$,
(ii) For $\delta \in[\widetilde{\delta}, \widetilde{\delta}]$, the manufacturer uses persistent predatory pricing (but not vertical collusion to exclude): $w_{1}^{E}(\delta)=\widetilde{w}_{1}(\delta)$ is decreasing with $\delta$ and predatory (i.e., below $c_{1}$ ), while $t_{1}^{E}(\delta)$ is increasing in $\delta$;
(iii) For $\delta \in[\widetilde{\tilde{\delta}}, 1]$, the manufacturer uses vertical collusion to exclude and a less predatory wholesale price: $w_{1}^{E}(\delta)=w_{12}(\delta)$ is increasing with $\delta$ (but still predatory) and converges back to $c_{1}$ as $\delta \rightarrow 1$, while, $t_{1}^{E}(\delta)$ is decreasing in $\delta$.

Figure 1 illustrates $w_{1}^{E}(\delta)$, the manufacturer's exclusionary wholesale price. We shall first describe the general intuition for how the discount factor affects the manufacturer's exclusionary strategies and then elaborate on each of the three relevant regions of the discount factor. For very low discount factors $(\delta \in[0, \widetilde{\delta}])$, the retailer is so impatient that the manufacturer does not need neither persistent predatory pricing nor vertical collusion to exclude. For very high discount factors ( $\delta \in[\widetilde{\delta}, 1]$ ) the retailer is so patient that vertical collusion to exclude (an implicit promise of a future reduction in the fixed fee) is used, so a less predatory wholesale price will do. For intermediate discount factors, $(\delta \in[\widetilde{\delta}, \widetilde{\delta}])$, the retailer is sufficiently patient to want to carry product 2 , but not patient enough to be profitably affected by a future promise of a reduced fixed fee. Hence, the only exclusionary tool here is persistent predatory pricing.

To further see the driving force behind the exclusionary mechanisms in each of the three regions of $\delta$, notice first that for $\delta \in[0, \widetilde{\delta}]$ when $w_{1}=c_{1}$, only selling 2 binds. In this range, the manufacturer does not apply any exclusionary tool.

For intermediate values of $\delta(\delta \in[\widetilde{\delta}, \widetilde{\widetilde{\delta}}])$, the manufacturer maximizes its exclusionary profits with persistent predatory pricing: $\widetilde{w}_{1}(\delta)<c_{1}$. Even though the wholesale price is persistently below cost, the manufacturer's overall profits can be higher than the manufacturer's profit from accommodating product 2 (as shown in the next subsection) thanks to the fixed fee. The goal of persistent predatory pricing here is to reduce the retailer's marginal profit from selling both products. In this range of $\delta$, the fee is $\underline{T}\left(\widetilde{w}_{1}(\delta)\right)=T_{2}\left(\widetilde{w}_{1}(\delta), \delta\right)=T_{12}\left(\widetilde{w}_{1}(\delta), \delta\right)$ and $\widetilde{w}_{1}(\delta)$ is a corner solution. As the retailer becomes more forward-looking within this range, the retailer has a stronger


Figure 1: The manufacturer's optimal $w_{1}, w_{1}^{E}(\delta)$, under exclusion as a function of $\delta$
incentive to offer product 2 alongside product 1, and the wholesale price becomes more predatory, while the fixed fee increases, to recoup losses. The tool of persistent predatory pricing does not depend on the assumption that the game is infinite. The manufacturer implements the same tool in a finite game. ${ }^{11}$

For high levels of $\delta(\delta \in[\widetilde{\widetilde{\delta}}, 1])$, the manufacturer combines a predatory (but less predatory) wholesale price $\left(w_{1}=w_{12}(\delta)\right)$ with vertical collusion to exclude. Now, the main exclusionary tool is the fixed fee $\left(t_{1}=T_{12}\left(w_{1}, \delta\right)\right)$. The manufacturer and retailer infinitely interact, so that if the retailer avoids product 2 in the current period, the manufacturer implicitly 'rewards' the retailer with a low fixed fee in future periods. The retailer knows that if it rejects the manufacturer's contract, it would improve product 2 and, starting from the next period, will not sell product 1 . Consequently, the retailer will not enjoy the reduction in the fixed fee. Recall that in this range of $\delta$, selling $1+2$ is binding, so the retailer compares the second term in the right-hand side of (2) to the left-hand side of (2). The fixed fee cancels out in the current period, but a reduced fixed fee enhances the left-hand side in future periods. Here, the wholesale price and fixed fee evolve in

[^9]opposite directions compared to the middle range of $\delta$ : The wholesale price becomes less predatory as $\delta$ increases (so as to reduce joint losses) while the fixed fee shrinks (and hence becomes more exclusionary), to exclude product 2 .

Put differently, a reduced fixed fee shrinks the retailer's long-term gain from improving product 2 , while persistent predatory pricing inflates the retailer's short-term sacrifice from improving product 2. Accordingly, vertical collusion to exclude gains more weight as $\delta$ increases, at the expense of persistent predatory pricing, because for low discount factors, it is more effective to inflate the retailer's short-term sacrifice from improving product 2 , via persistent predatory pricing, while for high discount factors it is more effective to diminish the retailer's long-term gain from improving product 2 , via vertical collusion to exclude. ${ }^{12}$

The more efficient tool of vertical collusion to exclude, available to patient parties, is unique to an infinite game. In the finite game, even for the high range of $\delta \in[\widetilde{\widetilde{\delta}}, 1]$, the manufacturer continues to use persistent predatory pricing as the sole exclusionary tool, with $w_{1}=\widetilde{w}_{1}(\delta) .{ }^{13}$ Note also that in our model, product 2 continues to be available to the retailer indefinitely, so that when the relationship between the manufacturer and the retailer is of an infinite duration as well, product 2 is always improvable by the retailer.

The result that persistent predatory pricing is part of the manufacturer's exclusionary strategy for any $\delta>\widetilde{\delta}$ stems from the fact that for such discount factors, the retailer's binding constraint is selling $1+2$. Then, by (2), the fixed fee does not affect the retailer's current profits when comparing between selling only product 1 and selling both products, because the retailer needs to pay this fee in both cases. The manufacturer cannot charge a different fixed fee when the retailer sells both products and when it sells only product 1 because this amounts to a type of explicit exclusive dealing, which we assume is banned by antitrust rules. Hence the only way to affect the retailer's constraint in the current period is lowering the wholesale price below cost, to intensify the retailer's shortterm loss from selling both products while product 2 is inferior. This exclusionary tool enables the supplier to exclude product 2 more profitably than when it uses only a future promise of a reduced

[^10]fixed fee, yet it sacrifices industry profits, by lowering the retail price below the monopoly price. For intermediate discount factors $(\delta \in[\widetilde{\delta}, \widetilde{\widetilde{\delta}}])$ satisfying the retailer's incentive compatibility constraint with equality via a corner solution $\left(w_{1}=\widetilde{w}_{1}(\delta)\right)$ is more profitable to the manufacturer, because in this range, $\widetilde{w}_{1}(\delta)>w_{12}(\delta)$ (see figure 1 ). This is while for high discount factors $(\delta \in[\widetilde{\widetilde{\delta}}, 1])$, the internal profit-maximizing wholesale price, of $w_{12}(\delta)$, is more profitable, since in this range $\widetilde{w}_{1}(\delta)<w_{12}(\delta)$. Naturally, the manufacturer prefers to exclude product 2 using the least predatory wholesale price.

By the intuition above, it is clear why the manufacturer does not use persistent predatory pricing nor vertical collusion to exclude when $\delta<\widetilde{\delta}$. Because in this range selling $1+2$ is not considered by the retailer, a predatory $w_{1}$ cannot affect the retailer's marginal profits from selling both products. The manufacturer implements the vertically integrated outcome with $w_{1}=c_{1}$, as depicted in Figure 1 , and sets $t_{1}=T_{2}\left(c_{1}, \delta\right)$. In this range, the fixed fee decreases with $\delta$, not as a future promise, but merely to adjust, in the current period, for the retailer becoming more patient ( $t_{1}$ equates the left-hand side of (2) to the first term on the right hand side, since selling 2 binds). Hence there is no vertical collusion to exclude in this range. The manufacturer does not reward the retailer in the next period for not selling product 2 in the current period. It merely charges a fixed fee in the current period that makes the retailer indifferent between selling only product 1 and selling only product 2. Indeed, the exclusionary contract in this range is the same even when the game is finite and there is no scope for a future reward of a reduced fixed fee. ${ }^{14}$

## The manufacturer's optimal accommodation contract

Suppose now that the manufacturer chooses to accommodate product 2 . The highest $t_{1}$ that the manufacturer can set needs to satisfy:

$$
\begin{equation*}
\pi_{12}^{R}\left(w_{1}\right)-t_{1}+\frac{\delta}{1-\delta} \pi_{2 H}^{V I} \geq \pi_{2}^{V I}+\frac{\delta}{1-\delta} \pi_{2 H}^{V I} . \tag{8}
\end{equation*}
$$

Hence, $t_{1}=\underline{T}\left(w_{1}\right)$. The manufacturer earns:

$$
\begin{equation*}
\Pi_{12}^{M}\left(w_{1}\right)=\left(w_{1}-c_{1}\right) \widehat{q}_{1}\left(w_{1}\right)+\underline{T}\left(w_{1}\right)=\pi_{12}^{V I}\left(w_{1}\right)-\pi_{2}^{V I} . \tag{9}
\end{equation*}
$$

[^11]Accordingly, under accommodation, the manufacturer sets $w_{1}=c_{1}$ to maximize $\pi_{12}^{V I}\left(w_{1}\right)$, and earns $\Pi_{12}^{M}\left(c_{1}\right)$. Then, in all future periods the retailer sells only the improved product 2 and the manufacturer earns zero.

## When does the manufacturer accommodate product 2?

We now ask whether the manufacturer chooses to exclude or to accommodate product 2. Comparing the manufacturer's profit under exclusion (equation (6) evaluated at $w_{1}^{E}$ ) and accommodation (equation (9) evaluated at $w_{1}=c_{1}$ ), we obtain the following result:

Proposition 2. (A vertically separated industry accommodates product 2 more than what maximizes industry profits) Under vertical separation, there is a unique cutoff $\delta^{V S}$ such that the market excludes (accommodates) product 2 if $\delta \leq \delta^{V S}\left(\delta>\delta^{V S}\right)$. Moreover, the market accommodates product 2 more than under vertical integration: $\delta^{V S}<\delta^{V I}$. In equilibrium:
(i) For $\delta \in[0, \widetilde{\delta}]$, the manufacturer excludes product 2 by setting $w_{1}=c_{1}$ and charging $t_{1}=$ $T_{2}\left(w_{1}, \delta\right)$ in all periods;
(ii) For $\delta \in\left[\widetilde{\delta}, \delta^{V S}\right]$, the manufacturer excludes product 2 by setting $w_{1}<c_{1}$ (and $t_{1}>0$ ), corresponding to the results of Proposition 1, in all periods;
(iii) For $\delta \in\left[\delta^{V S}, 1\right]$, the manufacturer accommodates product 2 by setting $w_{1}=c_{1}$ and $t_{1}=\underline{T}\left(c_{1}\right)$ in the first period. The retailer sells $1+2$ and then only the improved product 2 in all future periods.

Proposition 2 shows that a vertically separated market accommodates product 2 for values of $\delta$ under which a vertically integrated industry would exclude product 2 . This occurs for $\delta \in\left[\delta^{V S}, \delta^{V I}\right]$. The intuition for this result follows from Proposition 1 . When $\delta>\widetilde{\delta}$, excluding product 2 and, at the same time, maximizing industry profits (requiring $w_{1}=c_{1}$ ) is too costly for the manufacturer. Hence, the manufacturer finds it optimal to combine the two exclusionary tools, of persistent predatory pricing and vertical collusion to exclude. When $\delta$ approaches $\delta^{V S}$, these practices become prohibitively costly to the manufacturer, so it prefers to accommodate product 2 for discount factors below $\delta^{V I}$. This result indicates that under vertical separation, a market corresponding to
our model accommodates new products that improve over time more than what maximizes industry profits.

To highlight the role of the main feature of our model, namely that the new product improves if sold by the retailer, notice that persistent predatory pricing and vertical collusion to exclude cannot emerge absent this feature. The corollary below follows directly from the definition of $\widetilde{\delta}:{ }^{15}$

Corollary 1. (i) When the new product remains inferior even if sold by the retailer, the industry replicates the vertically integrated result for such a case and product 2 is always excluded with no need for explicit exclusive dealing. That is, $\widetilde{w}_{1}(\delta)>c_{1}$ (only selling 2 binds) and $w_{1}^{E}(\delta)=c_{1}$ for all $\delta \in[0,1]$.
(ii) When the new product is superior whether or not it is sold by the retailer, the industry replicates the vertically integrated result for such a case and the retailer sells only product 2 for all $\delta \in[0,1]$.

This corollary emphasizes how the framework we focus on, in which the retailer sacrifices shortterm profits by selling product 2 so as to improve it in the future, is what drives all of our results. Absent this framework, if product 2 remains inferior even if sold by the retailer, the Chicago School claim holds true: The parties replicate the vertically integrated outcome (no over-accommodation from an industry perspective), and exclude product 2 with a simple two part tariff, without requiring persistent predatory pricing, vertical collusion to exclude, or explicit exclusive dealing. Conversely, if product 2 is superior to product 1 regardless of whether it was sold by the retailer, the manufacturer needs to make negative profits in order to convince the retailer to hold its product, so the retailer sells only product 2, like a vertically integrated firm would. This, again, corresponds to the Chicago School claim that the manufacturer would not be able to exclude a superior product.

Another market-feature affecting our results is the size $q$, the minimum scale required to improve product 2. If $\underline{q}$ exceeds the monopoly quantity of product 1 , the retailer would never consider selling both products simultaneously. In such a case, the only binding constraint in (2) is selling 2 , and the parties can replicate the vertically integrated outcome without using persistent predatory pricing, vertical collusion to exclude, or explicit exclusive dealing. Another implication of the size of $\underline{q}$ is

[^12]that if it is smaller than the monopoly quantity of product 1 , but is large enough, vertical collusion to exclude does not emerge as an exclusionary tool. To see why, notice that whether exclusion in the infinite game via vertical collusion to exclude emerges in equilibrium or not depends on whether $\delta^{V S}>\widetilde{\widetilde{\delta}}$. In such a case, the manufacturer excludes product 2 with vertical collusion to exclude for $\delta \in\left[\widetilde{\widetilde{\delta}}, \delta^{V S}\right]$. Conversely, when $\delta^{V S}<\widetilde{\widetilde{\delta}}$, the more efficient exclusionary tool, of vertical collusion to exclude with a reduced fixed fee, is not used, since for all values of $\delta>\widetilde{\widetilde{\delta}}$, the manufacturer prefers to accommodate product 2 over excluding it. Therefore, we are more likely to observe vertical collusion to exclude when $\delta^{V S}$ is relatively high. That is, when exclusion remains profitable despite the retailer being relatively patient. Intuitively, this occurs when $\underline{q}$ is sufficiently large. In such a case, improving product 2 involves a higher sacrifice of short-term profits. Another case where this occurs is where the benefit from improving product 2, yielding $\pi_{2 H}^{V I}$ instead of $\pi_{1}^{V I}$, is relatively small. We can illustrate these intuitions using the following example.

## A linear demand example

Consider linear demand, which is initially $P(Q)=1-Q$ for both products, where $Q=q_{1}+q_{2}$. Suppose that $1>c_{2}>c_{1}>0$, such that $\pi_{i}^{V I}=\frac{\left(1-c_{i}\right)^{2}}{4}, i=1,2$. If in a certain period the retailer sells $q_{2} \geq \underline{q}$ (where $0<\underline{q}<\frac{1-c_{2}}{2}$ ), the profit from selling product 2 rises to $\pi_{2 H}^{V I}>\frac{\left(1-c_{1}\right)^{2}}{4}$.

Figure 2 shows the threshold values of $\delta^{V I}, \delta^{V S}, \widetilde{\widetilde{\delta}}$ and $\widetilde{\delta}$ as a function of $\underline{q}$ for a selected value of $\pi_{2 H}^{V I}$. The three shaded regions represent the parameter space in which the outcome under vertical separation diverges from the vertically integrated outcome. The first region (shaded blue) represents the case where $\delta \in\left[\delta^{V S}, \delta^{V I}\right]$. In this region, since $\delta<\delta^{V I}$, a vertically integrated monopoly would find it optimal to exclude product 2. Yet, because $\delta>\delta^{V S}$, the vertically separated industry accommodates product 2 . The other two regions in which the vertically separated outcome diverts from the vertically integrated one are the cases where $\delta \in\left[\widetilde{\tilde{\delta}}, \delta^{V S}\right]$ (the dotted red region) and $\delta \in[\widetilde{\delta}, \widetilde{\widetilde{\delta}}]$ (the striped green region). In these two regions product 2 is excluded under both vertical separation and vertical integration, but exclusion under vertical separation involves $w_{1}<c_{1}$, while exclusion under vertical integration involves $w_{1}=c_{1}$. Furthermore, for $\delta \in[\widetilde{\delta}, \widetilde{\widetilde{\delta}}]$ (the striped green region) the vertically separated industry excludes product 2 solely by using persistent predatory pricing: $w_{1}=\widetilde{w}_{1}(\delta)<c_{1}$. Conversely, for $\delta \in\left[\widetilde{\widetilde{\delta}}, \delta^{V S}\right]$ (the dotted red region), product 2 is excluded with the more efficient tool of vertical collusion to exclude with a reduced fixed fee, which enables


Figure 2: The threshold values $\delta^{V I}, \delta^{V S}, \widetilde{\widetilde{\delta}}$ and $\widetilde{\delta}$ (for $c_{1}=1 / 2, c_{2}=3 / 4$ and $\pi_{2 H}^{V I}=1 / 15$ )
the manufacturer to set $w_{1}=w_{12}(\delta)>\widetilde{w}_{1}(\delta)$. The figure further shows that the region in which the vertically separated industry uses vertical collusion to exclude vanishes when $\underline{q}$ is small enough (i.e., when $\underline{q}<\underline{q}^{\prime}$ ). The intuition for this result is that for low values of $\underline{q}$, it is difficult for the manufacturer to exclude product 2 , as the retailer only needs to sell a small number of units of the inferior product 2 in order to improve it. As a result, the region in which the manufacturer applies exclusionary strategies $\left(\delta \in\left[\widetilde{\delta}, \delta^{V S}\right]\right)$ is relatively narrow and applies only for low values of $\delta$, while the region in which the manufacturer accommodates (the inferior) product $2\left(\delta \in\left[\delta^{V S}, \delta^{V I}\right]\right)$ is wide and applies for high values of $\delta$. Because the region of strategic exclusion is narrow and applies only for low values of $\delta$, there is no scope for vertical collusion to exclude, which can only occur for high values of $\delta$ in which the retailer appreciates a future reward. Conversely, when $\underline{q}$ is high, it is easy for the manufacturer to exclude product 2 , so the region of strategic exclusion is wide and applies for high values of $\delta$. Here there is scope for vertical collusion to exclude.

It is also possible to show that when $\pi_{2 H}^{V I}$ is high, then the region of vertical collusion to exclude vanishes for all values of $\underline{q}$. Intuitively, the effect of $\pi_{2 H}^{V I}$ is opposite to the effect of $\underline{q}$. For high values of $\pi_{2 H}^{V I}$, it is difficult to exclude product 2 because the retailer expects to gain a high $\pi_{2 H}^{V I}$ by
improving product 2. Therefore, the region of strategic exclusion is narrow and applies only for low values of $\delta$, leaving no scope for vertical collusion to exclude. The opposite occurs when $\pi_{2 H}^{V I}$ is low.

Finally, the figure shows the two regions (in white) in which the manufacturer implements the vertically integrated outcome even under vertical separation. When $\delta \in[0, \widetilde{\delta}]$, the manufacturer excludes product 2 with $w_{1}=c_{1}$ and hence implements the vertically integrated outcome. Likewise, when $\delta \in\left[\delta^{V I}, 1\right]$, the manufacturer accommodates product 2 with $w_{1}=c_{1}$ as occurs under vertical integration.

## IV. ALLOWING EXCLUSIVE DEALING OR BANNING PERSISTENT PREDATORY

## PRICING

In this section we study the effect of allowing the manufacturer to impose exclusive dealing that makes contractual terms depend on whether the retailer sells product 2 along side product 1 (subsection IV(i)), and the effect of placing a ban on persistent predatory pricing (subsection IV(ii)).

IV(i). Exclusive dealing
Suppose now that the manufacturer is allowed by antitrust rules to explicitly impose exclusive dealing, by prohibiting the retailer from selling product 2 . The manufacturer offers a two-part-tariff contract (with or without an exclusive dealing clause) that is valid for the current period only. The retailer sells only product 1 if it accepts an exclusive dealing clause and sells only product 2 if it rejects the clause. In this sense, what we model, for simplicity, is an exclusive dealing clause under which if the retailer rejects it, it can sell only product 2 . An equivalent contract is a pair of two part tariffs, one that applies when the retailer does not sell product 2 and another one that applies in case the retailer sells both products. In Online Appendix C, we show that the parties implement the vertically integrated outcome for $\delta>\widetilde{\delta}$ by the manufacturer lowering the fixed fee to $\pi_{1}^{V I}-(1-\delta) \pi_{2}^{V I}-\delta \pi_{2 H}^{V I}$ instead of $\pi_{12}^{V I}-\pi_{2}^{V I}$ (and setting $w_{1}=c_{1}$ ) if the retailer avoids product 2 in that period. ${ }^{16}$

The retailer agrees to an exclusive dealing clause if:

$$
\begin{equation*}
\frac{\pi_{1}^{R}\left(w_{1}\right)-t_{1}}{1-\delta} \geq \pi_{2}^{V I}+\frac{\delta}{1-\delta} \pi_{2 H}^{V I} \tag{10}
\end{equation*}
$$

[^13]Hence, under exclusive dealing, the manufacturer sets $t_{1}=T_{2}\left(w_{1}, \delta\right)$ and earns:

$$
\begin{equation*}
\Pi_{1}^{M, E D}\left(w_{1}\right)=\left(w_{1}-c_{1}\right) q_{1}\left(w_{1}\right)+T_{2}\left(w_{1}, \delta\right)=\pi_{1}^{V I}\left(w_{1}\right)-\pi_{2}^{V I}-\delta\left(\pi_{2 H}^{V I}-\pi_{2}^{V I}\right) \tag{11}
\end{equation*}
$$

Hence the manufacturer sets $w_{1}=c_{1}$ so as to maximize joint profits. Comparing the manufacturer's profits under the exclusive and the non-exclusive contracts described in Proposition 2, we have:

Proposition 3. (exclusive dealing implements the vertically-integrated outcome) Suppose that the manufacturer can impose exclusive dealing. Then, for $\delta \in[0, \widetilde{\delta}]$, exclusive dealing is redundant, so the manufacturer excludes product 2 by offering $w_{1}=c_{1}$ and $t_{1}=T_{2}\left(c_{1}, \delta\right)$. For $\delta \in\left[\widetilde{\delta}, \delta^{V I}\right]$, the manufacturer imposes explicit exclusive dealing and again sets $w_{1}=c_{1}$ and $t_{1}=T_{2}\left(c_{1}, \delta\right)$. For $\delta \in\left[\delta^{V I}, 1\right]$, the manufacturer accommodates product 2 and sets $w_{1}=c_{1}$ and $t_{1}=\underline{T}\left(c_{1}\right)$.

Proposition 3 shows that when the manufacturer can impose exclusive dealing, it does so in a way that implements the vertically integrated outcome. The explicit prohibition to sell product 2 allows the manufacturer to set the wholesale price that maximizes industry profits, $w_{1}=c_{1}$, regardless of $\delta$, without the concern that the retailer would accept the manufacturer's contract and then also sell product 2. Hence, the manufacturer's and retailer's joint profits are the same as a vertically integrated firm, which is maximized by accommodating product 2 only if $\delta>\delta^{V I}$.

## IV(ii). Exclusion when persistent predatory pricing is prohibited

Suppose now that the manufacturer is not allowed to set a wholesale price below marginal costs. Then product 2 is accommodated for an even wider set of discount factors, while the retail price rises to the monopoly level:

Corollary 2. (Exclusion when persistent predatory pricing is prohibited) Suppose that the manufacturer cannot set a wholesale price below marginal costs. Then, the market accommodates product 2 for lower values of $\delta$ than when a below-cost wholesale price is allowed. There is a threshold, $\widetilde{\delta}^{V S}$, such that the manufacturer sets $w_{1}=c_{1}$ and the retailer avoids product 2 for $\delta \in\left[0, \widetilde{\delta}^{V S}\right]$ and sells products $1+2$ for $\delta \in\left[\widetilde{\delta}^{V S}, 1\right]$. Moreover, $\widetilde{\delta}<\widetilde{\delta}^{V S}<\delta^{V S}$.

This result can be easily demonstrated via Figure 1. If the wholesale price is forced not to be lower than $c_{1}$, the manufacturer cannot charge the persistently predatory price that would maximize
its exclusionary profits for $\delta>\widetilde{\delta}$. When $w_{1}=c_{1}$, the only exclusionary tool left to the manufacturer is vertical collusion to exclude via reduction of the fixed fee. But for $\delta>\widetilde{\delta}$, using such an exclusionary tool yields less exclusionary profits than when persistent predatory pricing is allowed. ${ }^{17}$ Corollary 2 also demonstrates how the divergence between the vertically integrated and vertically separated outcomes does not hinge on persistent predatory pricing. It hinges on the fact that when explicit exclusive dealing is prohibited, the manufacturer must charge the same fixed fee whether the retailer sells only product 1 or both products, so the fixed fee cannot affect the retailer's short-term sacrifice in improving product 2. Obviously, banning persistent predatory pricing in our model raises the retail price to monopoly levels, since $w_{1}=c_{1}$ and the retailer consequently sets the monopoly retail price.

## V. Welfare analysis and antitrust implications

V(i). Exclusive dealing, analogous practices, and persistent predatory pricing
We start by considering exclusive dealing and persistent predatory pricing. Our results can shed a new light on antitrust cases including explicit exclusive dealing and analogous practices, such as Broadcom's attempt to explicitly exclude competing chips used for the delivery of subscription video and broadband internet service. In July 2021, the U.S. FTC has reached a consent agreement with Broadcom. According to the FTC, in order to deter entry of new firms, Broadcom imposed exclusivity and tying on OEMs and service providers that deterred them from buying from Broadcom's rival's. This, according to the FTC, excluded 'low priced, nascent rivals' and impeded their product development efforts. The FTC held that these rivals ' ... could, by working with key OEMs and Service Providers, become stronger, more effective competitors' so as to pose 'competitive threats to [Broadcom's] monopoly power'. The FTC further concluded that Broadcom's practices impeded the efforts of leading Service Providers who 'sought to provide opportunities for capable but less established suppliers to gain experience and scale. ${ }^{18}$ This factual background implies that OEMs and

[^14]Service Providers needed to bear a short-term loss in order to give Broadcom's rivals an opportunity to improve. Our results imply that once such explicit exclusive dealing is banned, Broadcom may engage in persistent predatory pricing and vertical collusion to exclude. To the extent such practices are allowed, over-accommodation of the new product may result, but prices will fall. Similarly, in the Meritor case, Eaton, the dominant supplier of heavy duty truck transmissions, induced customers to buy over $90 \%$ of their requirements only from it. The rival, who sold a technically superior transmission, consequently exited the market, because it needed a market share of at least $10 \%$ to remain viable. The facts of the case imply that customers may have needed to bear a short-term loss from using Meritor's transmissions at a larger scale, as they involved a technology not used previously in the U.S, coupled with interim quality issues, so that adaptation was required. ${ }^{19}$ Again, our results imply that when such explicit exclusion is banned, persistent predatory pricing may emerge. This presents a theory of harm overlooked by the Meritor court. On the other hand, over-accommodation of the new product could also occur, implying Eaton's practice may have had ambiguous welfare effects - a consideration also overlooked by the court. In McWane, Inc. v. FTC, ${ }^{20}$ Star, a new manufacturer of pipe fittings, tried to compete with the dominant firm, McWane, but it started by contracting with foundries that produced raw casings for it, with plans to acquire a foundry of its own once it reached a critical scale of operations. The court held that ' $[\mathrm{w}]$ ithout a foundry of its own with which to manufacture fittings, Star was forced to settle for a 'more costly and less efficient' arrangement on account of higher shipping, labor, and logistical costs; smaller batch sizes; less specialized equipment.' McWane induced most of the customers to buy exclusively from it, and the court held that this kept Star's product 'below the critical level necessary ... to pose a real threat to [McWane's] monopoly'. Here too our results imply that in such a dynamic setting, where the new product is improvable, such explicit exclusive dealing raises prices, on one hand, but has ambiguous welfare effects regarding accommodation of the new (initially inferior) product, on the other. ${ }^{21}$ Note that in our framework, it suffices to exclude product 2 partially, by preventing it from

[^15]reaching the minimum scale of $\underline{q}$. This was what happened in both the Meritor and the McWane cases, in which the dominant firm's exclusionary contracts denied the entrant the efficient scale it required in order to improve.

For an example of persistent predatory pricing, courts have dismissed several antitrust suits against IBM for practices analogous to persistent predatory pricing of its established storage devices. Large customers who installed IBM's mainframe computers could use IBM's own storage devices, or rival storage devices that plug into IBM's system. The rival storage devices are often cheaper and perform better. Yet, the customer may face a trade-off between using IBM's own well-known and established storage device or taking a chance with a relatively new storage device. IBM's practice was to charge an allegedly below-cost price for its own storage devices, while raising the price of its CPU's. Since customers always required IBM's CPU's, this is analogous to the fixed fee accompanying the below-cost wholesale price in our model. This practice was consistently held by antitrust courts not to be illegal predatory pricing, since the total price IBM charged for its CPU and storage was above cost. ${ }^{22}$

In the context of our paper, an antitrust court or agency has three regimes to consider:
(i) Allowing exclusive dealing ('ED'), as in subsection IV(i)
(ii) Banning exclusive dealing but allowing persistent predatory pricing ('No ED,Yes pred'), as in Section III; and
(iii) Banning both exclusive dealing and persistent predatory pricing ('No ED, No pred'), as in subsection IV(ii) $)^{23}$

The following table summarizes the outcomes of the three regimes:
an incumbent drug manufacturer gaining explicit exclusivity from customers to exclude a rival selling a competing drug, where the customers have switching costs (see, e.g., Adamson 2017; Eisai, Inc. v. Sanofi Aventis U.S., LLC (3d Cir.) 821 F.3d 394 (2016)).
${ }^{22}$ Here, unlike in our model, the manufacturer sells two products, so rather than recouping the losses via a fixed fee, it recouped them via the price of the CPU's. See, e.g., Telex, California Computer Products Inc. v. International Business Machines (10th Cir.) 613 F2d 727 (1975); ILC Peripherals Leasing Corp. v. International Business Machines Corp. (United States District Court for the Northern District of California) 458 F. Supp. 423 (1978) and California Computer Products, Inc. v. International Business Machines Corp. (9th Cir.) 613 F.2d 727 (1979).
${ }^{23}$ When exclusive dealing is allowed, it matters not in our model whether persistent predatory pricing is allowed or not, because the manufacturer would not want to charge a predatory price if exclusive dealing is allowed.

| Regime |  | $\delta \in[0, \widetilde{\delta}]$ | $\delta \in\left[\widetilde{\delta}, \widetilde{\delta}^{V S}\right]$ | $\delta \in\left[\widetilde{\delta}^{V S}, \delta^{V S}\right]$ | $\delta \in\left[\delta^{V S}, \delta^{V I}\right]$ | $\delta \in\left[\delta^{V I}, 1\right]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (i) | ED | $w_{1}=c_{1}$ <br> exclusion |  |  |  | $w_{1}=c_{1}$ <br> accommodation |
| (ii) | No ED, Yes pred | $w_{1}=c_{1}$ <br> exclusion |  |  |  | $=c_{1}$ <br> odation |
| (iii) | No ED, <br> No pred |  |  |  | $w_{1}=c_{1}$ <br> accommodatio |  |

Table 1: The three antitrust regimes

As shown in Table 1, in the bottom range of $\delta([0, \widetilde{\delta}])$ and in the top range $\left(\left[\delta^{V I}, 1\right]\right)$ the three antitrust regimes are equivalent. The differences between them lie in the middle ranges, of $\widetilde{\delta}<\delta<\delta^{V I}$. In what follows, we focus our comparison on these ranges.

The table reveals that the welfare implications of our results can be assessed along two dimensions: the first dimension concerns the quantity the industry sells (and the corresponding price). In other words, does the industry supply a monopoly quantity, or is the monopoly distortion alleviated? The second dimension is whether the inferior (but improvable) product 2 is accommodated by the industry too much or too little from a welfare perspective.

The importance of these two dimensions in a particular regime depends, among other things, on the firms' discount factor. Let us focus on the welfare comparison between 'No ED, Yes pred' and 'ED'. ${ }^{24}$ When $\delta \in\left[\widetilde{\delta}, \delta^{V S}\right]$, ED decreases social welfare. This is because in this interval, product 2 is excluded with or without ED (recall that $\delta^{V S}<\delta^{V I}$, so for $\delta \in\left[\widetilde{\delta}, \delta^{V S}\right]$, product 2 is excluded under both regimes). However, the wholesale price is lower without ED, so the monopoly distortion is alleviated.

Consider now the range $\delta \in\left[\delta^{V S}, \delta^{V I}\right]$. Here, there is accommodation without ED and exclusion with ED. In both cases, the wholesale price equals marginal cost, so the welfare analysis hinges only on the second dimension, of whether product 2 is over-accommodated or over-excluded. Interestingly, in this interval, ED may increase social welfare. This can occur when in the absence of ED,

[^16]there is over accommodation from a social perspective. This depends on the parameters of the case at hand.

Let $S W_{1}\left(w_{1}\right)$ denote per-period social welfare as a function of $w_{1}$ when the retailer sells only product 1 and $S W_{12}\left(w_{1}\right)$ denote social welfare in a period in which the retailer sells both products. We have:

$$
S W_{1}\left(w_{1}\right)=\int_{0}^{q_{1}\left(w_{1}\right)}\left(p(q)-c_{1}\right) d q, \quad S W_{12}\left(w_{1}\right)=\int_{0}^{q_{1}\left(w_{1}\right)} p(q) d q-c_{2} \underline{q}-c_{1}\left(q_{1}\left(w_{1}\right)-\underline{q}\right) .
$$

It is socially optimal to accommodate product 2 when $\delta>\delta^{S W}$, where $\delta^{S W}$ is the solution to:

$$
\begin{equation*}
S W_{12}\left(c_{1}\right)+\frac{\delta}{1-\delta} S W_{2 H} \geq \frac{S W_{1}\left(c_{1}\right)}{1-\delta} \tag{12}
\end{equation*}
$$

and $S W_{2 H}$ denotes per-period social welfare when the retailer sells the improved product 2. It is always the case that $\delta^{S W}<\delta^{V I}$, because the vertically integrated monopoly (and, analogously, the manufacturer, in the case of ED) do not internalize the positive effect that improving product 2 has on consumers. However, when $\delta^{V S}<\delta^{S W}<\delta^{V I}$, absent ED, there is an interval, $\delta \in\left[\delta^{V S}, \delta^{S W}\right]$ in which the industry over-accommodates product 2 . Intuitively, the vertically separated industry bears the cost of inducing exclusion, while this does not involve a social cost, so it may overaccommodate an initially inferior product. In such cases, ED is welfare enhancing. To illustrate, let us return to our linear demand example. Assuming costs are linear after product 2 improves, we can express $S W_{2 H}$ as a function of $\pi_{2 H}^{V I}$ so that $S W_{2 H}=\frac{3}{2} \pi_{2 H}^{V I}$. Figure 3 depicts $\widetilde{\delta}, \delta^{V I}, \delta^{V S}$, and $\delta^{S W}$ as a function of $\underline{q}$ for two selected values of $\pi_{2 H}^{V I}$. In panel (a), when $\pi_{2 H}^{V I}$ is small, for all levels of $\underline{q}: \widetilde{\delta}<\delta^{V S}<\delta^{S W}<\delta^{V I}$. For $\delta \in[0, \widetilde{\delta}]$ and for $\delta \in\left[\delta^{V I}, 1\right]$, ED has no effect on welfare and for $\delta \in\left[\widetilde{\delta}, \delta^{V S}\right]$ and $\delta \in\left[\delta^{S W}, \delta^{V I}\right]$, ED is welfare-decreasing. However, in the blue-shaded region, $\delta \in\left[\delta^{V S}, \delta^{S W}\right]$, so that ED is welfare enhancing. Conversely, panel (b) depicts the case of a larger $\pi_{2 H}^{V I}$, in which, for low levels of $\underline{q}, \delta^{S W}>\delta^{V S}$, but for high levels of $\underline{q}, \delta^{S W}<\delta^{V S}$. Here, the blue-shaded region, in which ED is welfare-enhancing, shrinks, and disappears for large levels of q. It can similarly be shown that as $\pi_{2 H}^{V I}$ increases further (above $\pi_{2 H}^{V I} \cong 0.11$ ), the region in which ED is welfare-enhancing vanishes. The intuition for a lower $\pi_{2 H}^{V I}$ to broaden the region in which

ED is welfare enhancing is that the smaller is $\pi_{2 H}^{V I}$, the larger the scope for over-accommodation of product 2 absent ED. A small $\pi_{2 H}^{V I}$ implies that improving product 2 is not very welfare-enhancing. The manufacturer's incentives are not aligned with social welfare maximization, so absent ED, it accommodates product 2 for its own reasons-to reap a larger portion of the smaller pie. The intuition for the subtle effect of $\underline{q}$ is that as $\underline{q}$ increases, as illustrated in panels (a) and (b), all of the threshold values of $\delta\left(\delta^{S W}, \delta^{V S}\right.$ and $\left.\delta^{V I}\right)$ increase: It becomes more costly to sell inferior product 2, both from the manufacturer's and the social planner's perspective. For high $\pi_{2 H}^{V I}$, an increase in $\underline{q}$ causes the region in which ED is welfare-enhancing to vanish. The supplier better internalizes the welfare implications of a high $\underline{q}$ than those of a low $\pi_{2 H}^{V I}$. Recall from (9) that a high $\underline{q}$ diminishes the manufacturer's profit from accommodating product 2, while $\pi_{2 H}^{V I}$ has no effect on these profits. Since the manufacturer's product is not 'must have', it cannot fully internalize the social gain of accommodation improving product 2 , but it does internalize the social loss from selling product 2 when it is inferior.

To summarize, within the range of $\delta$ in which exclusive dealing makes a difference ( $\delta \in\left[\widetilde{\delta}, \delta^{V I}\right]$ ), in the low range, of $\delta \in\left[\widetilde{\delta}, \delta^{V S}\right]$, exclusive dealing is socially harmful, due to its price-increasing effect. In the high range, of $\delta \in\left[\delta^{S W}, \delta^{V I}\right]$, exclusive dealing is again socially harmful, due to the over-exclusion of product 2 . In the middle range, of $\delta \in\left[\delta^{V S}, \delta^{S W}\right]$, exclusive dealing is welfareenhancing. In our linear demand example, with linear costs, the circumstances under which this occurs involve a relatively low $\pi_{2 H}^{V I}$.

Focusing now on the low range, of $\delta \in\left[\widetilde{\delta}, \delta^{V S}\right]$, we have seen that if persistent predatory pricing is allowed, exclusive dealing is socially detrimental, in the sense that it raises the retail price to monopoly levels. Persistent predatory pricing helps alleviate the monopoly distortion, by reducing the retail price below the monopoly price. ${ }^{25}$ Notably, current U.S. antitrust case law implies that the persistent predatory pricing arising in our framework would not be a violation. Exclusion here is achieved via pricing alone, with no explicit referral as to whether product 2 is sold by the retailer. In such scenarios, the case law implies that the practice is legal if, overall, prices are above cost. ${ }^{26}$

[^17]

Figure 3: The threshold values $\widetilde{\delta}, \delta^{V I}, \delta^{V S}$, and $\delta^{S W}$ (for $c_{1}=1 / 2, c_{2}=3 / 4$ )

Indeed, in our model, the manufacturer of product 1 uses persistent predatory pricing only when its profits outweigh its (positive) profits from accommodation, and hence, overall, the manufacturer makes a positive per-period profit.

It is often claimed in the antitrust case law and legal literature that exclusive dealing induces pro-competitive price discounts. ${ }^{27}$ We show the contrary: in our model, for $\delta \in\left[\widetilde{\delta}, \delta^{V S}\right]$, when exclusive dealing is banned, the manufacturer excludes via persistent predatory pricing, reducing the retail price below the monopoly price. When exclusive dealing is allowed, the manufacturer raises the wholesale price and the retail price equals the monopoly price, because the manufacturer can exclude solely via the exclusive dealing agreement. Antitrust courts especially stress their notion that exclusive dealing reduces prices when the period of exclusivity is relatively short. They consistently hold that short-term exclusive deals can encourage competitors 'to improve the ... prices they offer in order to secure the exclusive positions. ${ }^{28}$ Yet, in our framework, exclusive dealing agreements are equally harmful and price-increasing when they are of a short duration. In our model, exclusivity is for only one period. It is the dynamic exclusionary equilibrium that induces the parties to keep renewing it.

## V(ii). Vertical mergers

Our analysis similarly provides antitrust implications for vertical mergers. Suppose that we apply regime (ii) (exclusive dealing is banned and persistent predatory pricing is allowed) to the vertically separated industry and recall that our results carry over to the case where an independent and strategic manufacturer sells product 2 .

Consider first a vertical merger between the retailer and the manufacturer of product 1 . This merger would shift the industry to the behavior corresponding to regime (i). That is, it would be as if we allowed exclusive dealing: The effects of a vertical merger on over or under-accommodation of product 2 are socially ambiguous and depend on the circumstances of each case. Just like exclusive dealing, a vertical merger excludes product 2 more than the social optimum, but absent

[^18]the vertical merger, the industry may over-accommodate product 2 from a social perspective. ${ }^{29}$ However, our results imply that a vertical merger between the established manufacturer and the retailer involves a price-hike for any $\delta \in\left[\widetilde{\delta}, \delta^{V S}\right]$, because the manufacturer no longer needs to engage in persistent predatory pricing, so monopoly pricing is restored. Conventional wisdom is that vertical mergers tend to cause price-reductions, in the sense that they eliminate double margins. But in our framework, there is no double margin under vertical separation. On the contrary, under regime (ii), the manufacturer charges a below-cost wholesale price, and this helps reduce the industry's monopoly distortion.

Consider now a vertical merger between the manufacturer of product 2 and the retailer. It is straightforward that the results of Section III, of vertical separation between the manufacturer of product 1 and the retailer, carry over to this case. In particular, Section III assumes that the retailer obtains product 2 at marginal cost, so it behaves as if it is integrated with the manufacturer of product 2 . The same outcome emerges when the manufacturer of product 2 is independent and strategic. In particular, even when the manufacturer of product 2 and the retailer are vertically separated, as shown in Online Appendix A, the manufacturer of product 2 would not want to charge a wholesale price different than its marginal cost. This implies that in our framework, a vertical merger between the manufacturer of (the improvable) product 2 and the retailer has no effect on the industry outcome and affects (ex post) only the division of profits among the merging firms. Therefore, in our framework, a vertical merger between the manufacturer of the inferior (but improvable) product and the retailer is benign and deserves lenient antitrust treatment.

## VI. Extensions

## VI(i). Product 2 is supplied by a strategic player

Our base model assumes for simplicity that product 2 is available to the retailer at marginal costs. The current extension describes the case where product 2 is sold by a strategic player. Its conclusion is that our main results carry over to this case. We provide the detailed analysis in Online Appendix A, while focusing here on the main features of the equilibrium and the intuition for the results.

Suppose that there are two competing manufacturers: $M 1$ sells product 1 and $M 2$ sells product

[^19]2. The two manufacturers offer two-part tariff contracts, $\left(w_{1}, t_{1}\right)$ and $\left(w_{2}, t_{2}\right)$. Then, the retailer decides whether to accept one of them or both. Contract offers can be either simultaneous or sequential. In the latter case, M1 (the incumbent supplier) makes the offer slightly before M2 (the new entrant).

If the retailer accommodates product 2 and improves it, in any following period there is a unique equilibrium in which the two manufacturers charge wholesale prices equal to their marginal costs and fixed fees of $t_{1}=0$ and $t_{2}=\pi_{2 H}^{V I}-\pi_{1}^{V I}$. The retailer accepts only $M 2$ 's offer and earns $\pi_{2 H}^{V I}-t_{2}=\pi_{1}^{V I}, M 2$ earns $t_{2}=\pi_{2 H}^{V I}-\pi_{1}^{V I}$ and $M 1$ earns 0 . Notice that these are the equilibrium strategies following improvement of product 2 under both simultaneous contract offers and when M1 offers its contract before M2.

Turning to the game in the first period, consider first M2's strategy. Unlike M1, M2 has no incentive to completely exclude product 1 . This follows because in the first period, M2 prefers its (initially inferior) product to be sold in a volume not exceeding the quantity necessary to improve it. Hence, the most profitable entry strategy for $M 2$ is to sell the retailer a quantity $\underline{q}$ of product 2 and let the retailer sell product 1 along-side product 2. Doing so maximizes the joint profits of M2 and the retailer. This contrasts M1's strategy, which, in an exclusionary equilibrium, involves a below-cost wholesale price, to induce the retailer to refrain from (initially inferior) product 2 completely. Given that $M 2$ has no incentive to exclude product 1 completely, $M 2$ sets $w_{2}=c_{2}$, again, to maximize its and the retailer's joint profits. Indeed, as shown in Online Appendix A, M2 sets $w_{2}=c_{2}$ whether "selling 2 " or "selling $1+2$ " is binding, and regardless of $M 1$ 's strategy.

In an equilibrium in which product 2 is excluded, by the reasoning above, $M 2$ sets $w_{2}=c_{2}$. As for the fixed fee, $t_{2}$, it is set, in an exclusionary equilibrium, so that $M 2$ 's overall profits are zero. This follows because in an exclusionary equilibrium, $M 2$ fails to convince the retailer to hold product 2 even if $M 2$ sets, in the first period, a negative fixed fee that transfers to the retailer all of the potential future profits from improving product 2 that $M 2$ will collect from the retailer following the improvement of product 2. This negative fixed fee places the retailer in the same situation as it would be had it purchased product 2 from a competitive fringe, so the retailer earns the same profit as in our base model. This is M2's best response to M1's exclusionary offer weather M1 and M2 move simultaneously or sequentially.

Next, consider M1's exclusionary offer. Because $w_{2}=c_{2}$, and because $M 2$ 's best response leaves the retailer with the same profits as in our based model, M1's exclusionary offer is the same as in our base model: For any $\delta \leq \delta^{V C}$, M1 uses the two exclusionary strategies of persistent predatory pricing and vertical collusion to exclude, according to the same conditions as in Proposition 1. Hence, the exclusionary equilibrium identified in our main model carries over to the case of a strategic player selling product 2, whether the game is simultaneous or sequential.

When $\delta>\delta^{V C}$, in the main model, $M 1$ accommodates product 2. Yet, unlike in the main model, in which the retailer obtains product 2 from a competitive fringe, if $M 1$ accommodates product 2, a strategic $M 2$ exploits this by charging a fixed fee that extracts part of the retailer's profits from improving product 2. Hence, the retailer's profit when product 2 is accommodated are lower than when the retailer buys product 2 from a competitive fringe. Here, when $M 1$, due to its first-mover position in the market, plays before $M 2, M 1$ has two options: First, to set the exclusionary contract that we identified in our base model, which will induce $M 2$ to set the fixed fee appropriate to the exclusionary equilibrium and $w_{2}=c_{2}$. M1's second option is to set the accommodation contract, which will be answered by $w_{2}=c_{2}$ and $M 2$ 's enhanced accommodation fixed fee. In both cases, M1 earns the exclusion and accommodation profits that we identified in our base model, respectively. Hence $M 1$ accommodates product 2 iff $\delta>\delta^{V S}$. The only difference between this outcome and the accommodation equilibrium in the main paper is the division of profits between the retailer and $M 2$. Their joint profits are the same as in the main model, M1's accommodation profits are the same, and product 2 is accommodated, as in the main model.

Suppose now that the game between $M 1$ and $M 2$ is simultaneous. Here, for $\delta>\delta^{V C}$, there is no pure-strategy equilibrium. This follows because if $M 2$ offers its accommodation contract, with the larger fixed fee extracting some of the retailer's profits, $M 1$ would like to deviate to its exclusionary contract. This, however, does not significantly affect the generality of our results. First, the assumption that the manufacturer of product 1 plays before the manufacturer of product 2 is reasonable, given that $M 1$ is the incumbent supplier, which is likely to have a first-mover position in offering the retailer a contract before M2. Second, as stressed above, M1's exclusionary strategies, which are our main focus, are robust, either in a simultaneous or a sequential game. Note also that in any mixed strategy equilibrium of the simultaneous game, for any $\delta>\delta^{V C}$, there will
be over-accommodation of product 2, as identified by Proposition 2 of the paper, albeit with some probability, instead of with certainty. Third, any mixed strategy equilibrium is short-lived, in the sense that once the probability of accommodating product 2 is realized in a certain period, in all following periods, all players play a pure strategy. In this stationary pure-strategy equilibrium, as discussed above, the retailer sells only the improved product $2, M 1$ offers the retailer the contract $t_{1}=0$ and $w_{1}=c_{1}$ and makes no sales, and $M 2$ offers $t_{2}=\pi_{2 H}^{V I}-\pi_{1}^{V I}$ and $w_{2}=c_{2}$.

VI(ii). Finite game
In Online Appendix D , we solve for a finite game with $N+1$ periods, where $N \geq 1$. We show that vertical collusion to exclude with a reduced fixed fee as a tool mitigating the distortion from persistent predatory pricing, which we identified in Section III for $\delta>\widetilde{\widetilde{\delta}}$, never emerges in a finite game. ${ }^{30}$ This is so even when $N \rightarrow \infty$. Instead, for all values of $\delta>\widetilde{\delta}$ and $N \in[1, \infty]$, the manufacturer's only exclusionary tool is persistent predatory pricing. In particular, in order to exclude product 2 , the manufacturer sets $w_{1}=\widetilde{w}_{1}(\delta)<c_{1}$. This is the same persistent predatory price used by the manufacturer for intermediate levels of $\delta(\delta \in[\widetilde{\delta}, \widetilde{\widetilde{\delta}}])$ in the infinite-horizon case. Thus, for $\delta>\widetilde{\widetilde{\delta}}$, while exclusion in the infinite-horizon case is achieved via a lower fixed fee, and a less predatory wholesale price $\left(w_{12}>\widetilde{w}_{1}(\delta)\right)$, in the finite game it involves only the more predatory wholesale price $\widetilde{w}_{1}(\delta)$. This result indicates that in the infinite game, the exclusionary equilibrium for $\delta>\widetilde{\widetilde{\delta}}$ involving vertical collusion to exclude relies on trust between the retailer and the manufacturer that cannot exist when the game is expected to end at some point. It further implies that the threshold of $\delta$ above which the manufacturer accommodates product $2, \delta^{V S}$, is higher in the infinite game than in the finite game for $N \rightarrow \infty$. That is, it is more profitable for the manufacturer to exclude product 2 in an infinite game than in a finite game in which the number of periods approaches infinity.

Let us now highlight the welfare implications of a finite horizon. Vertical relations of a finite horizon may arise in practice. First, the relationship may last only for a given project, the period of which is known in advance. In dynamic and innovative industries, the horizon may be finite as well. Consider the above-mentioned IBM mainframe example. Although the customer expects to require peripheral products for its IBM system for many years, the horizon is finite: at some

[^20]point, the system will become obsolete and require replacement. Similarly, the Broadcom example cannot be expected to be of an infinite horizon. At some point the chips in question will become obsolete. Our results imply that explicit exclusive dealing, other explicit exclusionary tools, and vertical mergers raise prices more (and in this sense are more anticompetitive) when the horizon is finite: when exclusive dealing or a vertical merger are banned, the manufacturer needs to charge a more predatory (and welfare enhancing) price in order to exclude product 2, relative to the infinite horizon case. This result sheds a new light on the conventional wisdom, according to which in dynamic technological industries, exclusive dealing, tying, or vertical mergers are pro-competitive. ${ }^{31}$ While this may be true in other frameworks, it is not true in ours. In our framework, with an infinite horizon, the parties can behave more anticompetitively with regard to pricing even without exclusive dealing or a vertical merger, because they can build mutual trust. With a finite horizon, they lose this mutual trust, and then banning such practices makes more of a difference: absent exclusive dealing or a vertical merger, the result may be significantly more competitive with respect to price.

## VI(iii). Platform instead of retailer

In digital markets, online retail platforms such as Amazon, Apple and Google (Apple's AppStore and Google's Playstore serve as retail platforms that sell apps) typically charge commission fees that are a percentage of the revenue, while allowing the supplier to set the retail price. In this extension, we informally comment on how our results might be affected by such settings.

Suppose that the retailer is a platform, which charges the manufacturer of product 1 a commission $\alpha$ that is a percentage of the revenue. The manufacturer keeps the remaining revenue and charges the platform a fixed fee each period. The manufacturer also sets the retail price $p$. Accordingly, in the first stage, the manufacturer sets $\alpha$ and $p$. In the second stage, the platform decides whether to accept the manufacturer's contract and sell only product 1 , or sell product 2 , thereby improving it in the next period, similarly to our base model.

[^21]We expect our results to carry over to this case in the following sense: As in our main model, for low levels of $\delta$, the manufacturer can exclude product 2 while setting $p=p^{m}$ and $\alpha=0$, where $p^{m}$ is the monopoly retail price. Consider now higher levels of $\delta$, for which the platform's incentive to improve product 2 is too strong to accept a contract involving $p=p^{m}$ and $\alpha=0$. Unlike our main model, the manufacturer can exclude product 2 while retaining $p=p^{m}$, by raising $\alpha$. This inflates the platform's short-term sacrifice in improving product 2, because, by having to sell a quantity $\underline{q}$ of product 2, the platform loses on its share $\alpha$ of product 1's revenue. However, for larger levels of $\delta$, the exclusionary tool of increasing $\alpha$ is exhausted when, even for $\alpha=1$, the platform still prefers improving product 2. For such levels of $\delta$, the manufacturer of product 1 can use a second exclusionary tool, of reducing the retail price $p$. This too inflates the platform's losses from improving product 2 , because it reduces the price that the platform can charge for product 2 . Yet, as in our main model, this second exclusionary practice reduces industry profits, since $p<p^{m}$. A third exclusionary tool, similar to our main model, is reduction of the fixed fee. Such a reduction serves as a prize the platform expects to receive, in the next period, for not improving product 2 in the current period. We expect that here too, if the parties are not allowed to stipulate different contractual terms depending on whether the platform sells product 2 (i.e., they are not allowed to sign an explicit exclusive dealing contract), the outcome will diverge from what maximizes industry profits, in a way similar to our main model.

## VI(iv). Competing retailers

The paper assumes that there is a monopolistic retailer. This raises the question of how the results change when the manufacturer faces competing retailers.

Consider an extension to our base model with two competing retailers. Our results trivially carry over to the case where retailers are highly differentiated, such that each retailer faces an (almost) separate market. Instead, consider the opposite extreme, in which the two retailers are homogeneous and attract the same set of consumers. Suppose further that once a retailer sells a quantity $q$ of product 2 and improves it, product 2 is improved for both retailers. This corresponds to most of the relevant settings. For instance, if product 2 is improved via learning by consumers, then once consumes try product 2 sold by one retailer, they learn about it and are then willing to pay more for it even when they buy it from the second retailer. Likewise, if selling a quantity
$q$ of product 2 improves it by reducing its costs of production, then this cost improvement occurs regardless of the identity of the retailer who sells it.

Accordingly, with two homogeneous and identical retailers, they earn zero profits from selling only product 1, because Bertrand price competition dissipates their profits. Hence the manufacturers of both products 1 and 2 cannot charge retailers a positive fixed fee. Once product 2 is improved, retailers' profits from selling only product 2 are also zero, for the same reason. Therefore, assuming the manufacturer of product 2 cannot offer exclusivity to a retailer that improves its product, and cannot compensate the retailer for selling a quantity $\underline{q}$ of product 2 via a negative fixed fee, retailers will not want to improve product $2 .{ }^{32}$ They behave myopically and sell only product 1 if $w_{1}<c_{2}$ (or if $w_{1}<w_{2}$, when product 2 is sold by a strategic manufacturer) and sell only product 2 otherwise. The manufacturer of product 1 can therefore exclude product 2 indefinitely by charging $w_{1}=c_{2}$ and $T_{1}=0$. In order to motivate retailers to carry product 2 , a strategic manufacturer supplying product 2 needs to offer a retailer that improved product 2 slotting allowances or an exclusive dealership. We comment on such practices in the next subsection.

Absent such practices by a strategic manufacturer of product 2, the above-mentioned insights imply that as retailers become more homogeneous, it is easier for the manufacturer of product 1 to exclude product 2 via a simple two part tariff. This implies that more intense downstream competition has ambiguous welfare effects in a setting such as ours: while it can reduce overaccommodation of product 2 , it can also enhance its over-exclusion. Also, since the manufacturer of product 1 has less need for persistent predatory pricing, this, in itself, tends to increase retail prices.
$\mathrm{VI}(\mathrm{v})$. Vertical restraints imposed by the manufacturer of product 2
This subsection briefly discusses the contractual arrangements that a strategic manufacturer of product 2, M2, can impose in order to motivate the retailer to accommodate product 2.

Consider first the case of a monopolistic retailer. Recall that our base model reveals that when the manufacturer of product $1, M 1$, cannot impose exclusive dealing, there is over-accommodation of product 2, in comparison with the level of accommodation under vertical integration. M2 cannot

[^22]further improve the prospects of accommodation by paying the retailer a slotting fee, because our result holds for any two-part tariff set by $M 2$, including a negative fixed fee. Intuitively, suppose that $\delta<\delta^{V S}$ so that the retailer prefers to exclude product 2 . This occurs in our base model even when product 2 is available to the retailer at marginal costs and even when the retailer extracts all profits from selling the improved product 2 in all future periods. Hence, a strategic $M 2$ cannot profitably offer the retailer a fixed fee that will change the retailer's decision. Similarly unhelpful is an exclusive dealing contract imposed by $M 2$. An exclusive dealing contract forcing the retailer to sell only the inferior product 2 decreases $M 2$ and the retailer's joint profit compared to a contract motivating the retailer to sell only a quantity $\underline{q}$ of product 2 along side superior product 1 .

Suppose now that there are competing retailers. Here, a strategic $M 2$ can benefit from expanding the contract space, in at least two directions, on which we only briefly comment here. First, M2 can offer retailers slotting allowances. Recall from our discussion in the previous subsection that under competition between homogeneous retailers, they expect future profits from selling the improved product 2 to be zero and are reluctant to sell the inferior product 2. M2 can solve this problem by paying retailers a slotting fee in the first period in return for selling product 2. In future periods, once product 2 is improved, $M 2$ can cover the costs of the slotting fees by charging a wholesale price above cost. ${ }^{33}$ An alternative tool that $M 2$ can use is to grant one of the retailers an exclusive dealership. Accordingly, this retailer would be the only seller of the improved product 2 in the following periods. This retailer would have the incentive to incur the losses from selling the inferior product 2 in the first period.

The availability of such practices to $M 2$ restores $M 1$ 's incentive to use its own two part tariff, or explicit exclusive dealing, in order to exclude product 2. An open question is whether, absent exclusive dealing with $M 1$, the above-mentioned practices used by $M 2$ encourage accommodation at the same level as in the case of a monopolistic retailer, i.e., for $\delta>\delta^{V S}$. This could shed light on the antitrust implications of such practices used by $M 2$. We leave these questions for future research.

## VII. Conclusion

[^23]This article revealed that a vertically separated industry fails to maximize industry profits when a new product is improvable if sold by the retailer. This contrasts the conventional Chicago School argument that with fixed fees, a supplier and retailer will elect quantities, prices and exclusion decisions to maximize joint profits, and divide them via the fixed fees. The economic literature has dealt with the Chicago School claim by studying situations in which new products are excluded although they are superior or those in which exclusion harms welfare although new products are inferior. Instead, we study the common situation in which the new product is initially inferior, but if the retailer is willing to sell it and sacrifice short-term profits, it improves. This scenario affects vertical relations: The manufacturer excludes the new product with persistent predatory pricing, to distort the retailer's current marginal profit from selling both products, and with vertical collusion to exclude, to reward the retailer with an implicit future promise of a reduced fixed fee if the new product is not improved. With these two exclusionary strategies, the manufacturer takes advantage of the retailer's tradeoff of bearing short-term losses so as to improve the new product. But for a high enough discount factor, these exclusionary strategies become too costly to the manufacturer, so it accommodates the new product even though this implies no future sales for the manufacturer. This occurs for discount factors lower than the discount factor in which a vertically integrated firm owning both the manufacturer and the retailer would accommodate the new product. A vertical relationship of an indefinite horizon helps the parties shrink the divergence between the vertically integrated and vertically separated outcomes, because only then vertical collusion to exclude is available, so exclusion is achieved with a less predatory wholesale price. Yet, even then, the divergence between vertical integration and vertical separation persists. When the horizon of the vertical relationship is finite, the only exclusionary tool is persistent predatory pricing, and the industry departs from joint profit maximization even more than in the infinite-horizon case. The results expose a new theory of harm for exclusive dealing and vertical mergers, while showing how such practices may nevertheless have ambiguous welfare effects, by preventing over-accommodation of the new product. They also introduce welfare implications for the antitrust rules concerning persistent predatory pricing, and shed light on the policy implications of such practices in a dynamic, technologically changing, environment.

## Appendix

Below are the proofs of Lemmas' 1-2 and Propositions 1-4.

## Proof of Lemma 1:

Proof of parts (i) and (ii):

We start by showing that given $w_{1}$, either $T_{12}\left(w_{1}, \delta\right) \leq T_{2}\left(w_{1}, \delta\right)<\underline{T}\left(w_{1}\right)$ or $\underline{T}\left(w_{1}\right)<T_{2}\left(w_{1}, \delta\right) \leq$ $T_{12}\left(w_{1}, \delta\right)$. To this end, the gap $\underline{T}\left(w_{1}\right)<T_{2}\left(w_{1}, \delta\right)$ is:

$$
\begin{equation*}
\underline{T}\left(w_{1}\right)-T_{2}\left(w_{1}, \delta\right)=\delta\left(\pi_{2 H}^{V I}-\pi_{2}^{V I}\right)-\left(\pi_{1}^{R}\left(w_{1}\right)-\pi_{12}^{R}\left(w_{1}\right)\right), \tag{13}
\end{equation*}
$$

and it is straightforward to see that $\underline{T}\left(w_{1}\right)-T_{12}\left(w_{1}, \delta\right)=-\frac{1}{\delta}\left[T_{2}\left(w_{1}, \delta\right)-\underline{T}\left(w_{1}\right)\right]$. Hence, when $\underline{T}\left(w_{1}\right)-T_{2}\left(w_{1}, \delta\right) \geq 0\left(\underline{T}\left(w_{1}\right)-T_{2}\left(w_{1}, \delta\right)<0\right), T_{12}\left(w_{1}, \delta\right) \leq T_{2}\left(w_{1}, \delta\right)<\underline{T}\left(w_{1}\right)\left(\underline{T}\left(w_{1}\right)<T_{2}\left(w_{1}, \delta\right) \leq\right.$ $\left.T_{12}\left(w_{1}, \delta\right)\right)$.

Next, we show how each of these two possibilities affect the binding constraint on $t_{1}$. Recall that the binding constraint is selling 2 when $t_{1}>\underline{T}\left(w_{1}\right)$, and that if selling 2 binds, then to induce the retailer to avoid product $2, t_{1} \leq T_{2}\left(w_{1}, \delta\right)$. Hence in order to induce the retailer to avoid product 2 when its binding constraint is selling 2 , it must be that $\underline{T}\left(w_{1}\right)<t_{1} \leq T_{2}\left(w_{1}, \delta\right)$. Such a $t_{1}$ exists only if $\underline{T}\left(w_{1}\right)<T_{2}\left(w_{1}, \delta\right)$. This further implies that if $\underline{T}\left(w_{1}\right) \geq T_{2}\left(w_{1}, \delta\right)$, the binding constraint for an exclusionary equilibrium must be selling $1+2$. Recall that in this case, $T_{12}\left(w_{1}, \delta\right) \leq T_{2}\left(w_{1}, \delta\right)<$ $\underline{T}\left(w_{1}\right)$, so the manufacturer sets $t_{1}=T_{12}\left(w_{1}, \delta\right)$. Likewise, when $\underline{T}\left(w_{1}\right)<T_{2}\left(w_{1}, \delta\right)$, selling 2 binds and the manufacturer sets $t_{1}=T_{2}\left(w_{1}, \delta\right)$, because, as noted above, $\underline{T}\left(w_{1}\right)<T_{2}\left(w_{1}, \delta\right) \leq T_{12}\left(w_{1}, \delta\right)$.

Finally, we show how the comparison between $w_{1}$ and $\widetilde{w}_{1}(\delta)$ affects each of the two possibilities in parts (i) and (ii) of the lemma. To this end, recall that $\widetilde{w}_{1}(\delta)$ is the level of $w_{1}$ in which $\underline{T}\left(w_{1}, \delta\right)=T_{2}\left(w_{1}, \delta\right)=T_{12}\left(w_{1}, \delta\right)$, so at $w_{1}=\widetilde{w}_{1}(\delta), \underline{T}\left(w_{1}\right)-T_{2}\left(w_{1}, \delta\right)=0$. Notice also that the gap $\underline{T}\left(w_{1}\right)-T_{2}\left(w_{1}, \delta\right)$ is increasing with $w_{1}$ because, from the envelope theorem:

$$
\frac{\partial\left(\underline{T}\left(w_{1}\right)-T_{2}\left(w_{1}, \delta\right)\right)}{\partial w_{1}}=q_{1}\left(w_{1}\right)-\widehat{q}_{1}\left(w_{1}\right)>0,
$$

Moreover, $\underline{T}\left(w_{1}\right)-T_{2}\left(w_{1}, \delta\right)$ is increasing with $\delta$, because $\pi_{2 H}^{V I}>\pi_{2}^{V I}$. Accordingly, the condition $\underline{T}\left(w_{1}\right)-T_{2}\left(w_{1}, \delta\right)<0$, in which the binding constraint for an exclusionary equilibrium is selling 2 , is equivalent to $w_{1}<\widetilde{w}_{1}(\delta)$, and the condition $\underline{T}\left(w_{1}\right)-T_{2}\left(w_{1}, \delta\right) \geq 0$, in which the binding constraint for an exclusionary equilibrium is selling $1+2$, is equivalent to $w_{1} \geq \widetilde{w}_{1}(\delta)$.

Proof of part (iii): To study the features of $\widetilde{w}_{1}(\delta)$, recall that $\widetilde{w}_{1}(\delta)$ is the solution to $\underline{T}\left(w_{1}\right)-$ $T_{2}\left(w_{1}, \delta\right)=0$, where $\underline{T}\left(w_{1}\right)-T_{2}\left(w_{1}, \delta\right)$ is defined in (13). Starting with $\delta=0$, we have $\underline{T}\left(c_{2}\right)-$ $T_{2}\left(c_{2}, 0\right)=0$ because at $w_{1}=c_{2}, \pi_{1}^{R}\left(c_{2}\right)=\pi_{12}^{R}\left(c_{2}\right)$ and $\delta=0$. This implies that $\widetilde{w}_{1}(0)=c_{2}$. Since $\underline{T}\left(w_{1}\right)-T_{2}\left(w_{1}, \delta\right)$ is increasing with $w_{1}$ and $\delta, \widetilde{w}_{1}(\delta)$ is decreasing with $\delta$. Next we show that $\widetilde{w}_{1}(\widetilde{\delta})=c_{1}$. To this end, we have that at $w_{1}=c_{1}$ and $\delta=0, \underline{T}\left(c_{1}\right)-T_{2}\left(c_{1}, 0\right)=-\left(\pi_{1}^{R}\left(c_{1}\right)-\pi_{12}^{R}\left(c_{1}\right)\right)=$ $-\left(c_{2}-c_{1}\right) \underline{q}<0$. On the other hand, $\underline{T}\left(c_{1}\right)-T_{2}\left(c_{1}, \delta^{V I}\right)>0$ and $\underline{T}\left(c_{1}\right)-T_{2}\left(c_{1}, \delta\right)$ is increasing in $\delta$. Accordingly, there is a unique cutoff, $\widetilde{\delta}$, such that $\widetilde{w}_{1}(\widetilde{\delta})=c_{1}$.

Proof of part (iv): The level of $\delta$ above which a vertically integrated firm prefers selling $1+2$ is the solution to (after rewriting (1)):

$$
\begin{equation*}
\delta\left(\pi_{2 H}^{V I}-\pi_{12}^{V I}\right) \geq \pi_{1}^{V I}-\pi_{12}^{V I} . \tag{14}
\end{equation*}
$$

Likewise, under vertical separation, it follows from part (ii) that the level of $\delta$ above which the binding constraint is selling $1+2$ is the solution to:

$$
\begin{equation*}
\underline{T}\left(w_{1}\right) \geq T_{2}\left(w_{1}, \delta\right) \Longleftrightarrow \delta\left(\pi_{2 H}^{V I}-\pi_{2}^{V I}\right) \geq \pi_{1}^{R}\left(w_{1}\right)-\pi_{12}^{R}\left(w_{1}\right) . \tag{15}
\end{equation*}
$$

At $w_{1}=c_{1}$, the right hand side of (14) and (15) are identical (recalling that $\pi_{1}^{R}\left(c_{1}\right)=\pi_{1}^{V I}$, $\left.\pi_{12}^{R}\left(c_{1}\right)=\pi_{12}^{V I}\right)$. Yet, when $\delta>0$, the left-hand side of (15) is almost identical to the left-hand side of (14), except that the term $-\pi_{12}^{V I}$ from (14) replaces the term $-\pi_{2}^{V I}$ from (15). Since $\pi_{12}^{V I}>\pi_{2}^{V I}$, it is clear that at $w_{1}=c_{1}$, the left-hand side of (14) is lower. Hence if, under vertical separation, the manufacturer sets $w_{1}=c_{1}$, the retailer has an incentive to sell $1+2$ for discount factors below $\delta=\delta^{V I}$. These are discount factors in which a vertically integrated firm (with an implicit wholesale price of $w_{1}=c_{1}$ ) would prefer to sell only product 1 . Accordingly, recalling from part (iii) that $\widetilde{w}_{1}(\widetilde{\delta})=c_{1}$, it follows that $\widetilde{\delta}<\delta^{V I}$.

## Proof of Proposition 1:

We first establish the features of $w_{12}(\delta)$ (we established the features of $\widetilde{w}_{1}(\delta)$ in the proof of Lemma 1). Differentiating the term in the squared brackets of the second line of (6) with respect to $w_{1}$, using the envelope theorem and recalling that $\widehat{q}_{1}\left(w_{1}\right)=q_{1}\left(w_{1}\right)-\underline{q}$, we have that $w_{12}(\delta)$ is the solution to:

$$
\begin{equation*}
\left.\frac{d \Pi\left(w_{1}\right)}{d w_{1}}\right|_{w_{1} \geq \widetilde{w}_{1}(\delta)}=\frac{\partial \pi_{1}^{V I}\left(w_{1}\right)}{\partial w_{1}}-\frac{1-\delta}{\delta} \underline{q} . \tag{16}
\end{equation*}
$$

The following lemma derives the features of $w_{12}(\delta)$ :

Lemma. At $\delta \rightarrow 0, w_{12}(\delta) \rightarrow-\infty ; w_{12}(\delta)$ is increasing in $\delta$ and $w_{12}(\delta)=c_{1}$ at $\delta=1$.

Proof: For $\delta=1$, the second term in (16) vanishes, hence $w_{12}(\delta)=c_{1}$. For any $\delta<1, w_{12}(\delta)<c_{1}$. To see why, evaluating (16) at $w_{12}(\delta)=c_{1}$, the first term vanishes, but the term in the squared brackets is negative, hence (16) is negative. Moreover, $w_{12}(\delta)$ is increasing with $\delta$ because $\frac{1-\delta}{\delta}$ is decreasing with $\delta$ and $\pi_{1}^{V I}\left(w_{1}\right)$ is concave. Finally, since the second term in (16) is negative, it is clear why when $\delta \rightarrow 0, w_{12}(\delta) \rightarrow-\infty$.

This completes the proof of the intermediate Lemma. ${ }^{34}$

Going back to the proof of Proposition 1, we have from the features of $\widetilde{w}_{1}(\delta)$ and $w_{12}(\delta)$ that there are two cutoffs $\widetilde{\delta}$ (the solution to $\widetilde{w}_{1}(\delta)=c_{1}$ ) and $\widetilde{\tilde{\delta}}$ (the solution to $w_{12}(\delta)=\widetilde{w}_{1}(\delta)$ ). For $\delta \in[0, \widetilde{\delta}]$, $\widetilde{w}_{1}(\delta)>c_{1}>w_{12}(\delta)$. In this range, the manufacturer maximizes the first line of (6) and sets $w_{1}=c_{1}$. For $\delta \in[\widetilde{\delta}, \widetilde{\widetilde{\delta}}], c_{1}>\widetilde{w}_{1}(\delta)>w_{12}(\delta)$. In this range, if the manufacturer sets $w_{1}=c_{1}$, the retailer deviates from the exclusionary equilibrium and sells $1+2$. Also, the manufacturer in this range can do better than setting $w_{1}=w_{12}(\delta)$ by setting a higher wholesale price of $w_{1}=\widetilde{w}_{1}(\delta)$ instead. In this corner solution, both the retailer's constraints bind, as $t_{1}=T\left(w_{1}, \delta\right)=T_{2}\left(w_{1}, \delta\right)=\underline{T}\left(w_{1}\right)$. Finally, for $\delta \in[\widetilde{\tilde{\delta}}, 1], c_{1}>w_{12}(\delta)>\widetilde{w}(\delta)$, so the manufacturer maximizes the second line of (6) by setting $w_{1}=w_{12}(\delta)$. This establishes $w_{1}^{E}(\delta)$ as the $w_{1}$ that maximizes (6). Note that following the above definitions of $\widetilde{\delta}$ and $\widetilde{\widetilde{\delta}}, w_{1}^{E}(\delta)$ is continuous in $\delta$. The manufacturer's exclusionary

[^24]profit (6) too is continuous in $\delta$. In particular, at $w_{1}=\widetilde{w}_{1}(\delta)$, it is continuous as at this point $t_{1}=T\left(w_{1}, \delta\right)=T_{2}\left(w_{1}, \delta\right)=\underline{T}\left(w_{1}\right)$.

The effect of $\delta$ on $w_{1}^{E}(\delta)$ follows from the proof of Lemma 1 and the lemma above. Turning to the effect of $\delta$ on $t_{1}$, we have that for $\delta \in[0, \widetilde{\delta}], t_{1}=T_{2}\left(c_{1}, \delta\right)=\pi_{1}^{V I}-\pi_{2}^{V I}-\delta\left(\pi_{2 H}^{V I}-\pi_{2}^{V I}\right)$ which is decreasing with $\delta$. For $\delta \in[\widetilde{\delta}, \widetilde{\widetilde{\delta}}], t_{1}=\underline{T}\left(\widetilde{w}_{1}(\delta)\right)=\pi_{12}^{R}\left(\widetilde{w}_{1}(\delta)\right)-\pi_{2}^{V I}$ which is increasing in $\delta$ because $\pi_{12}^{R}\left(w_{1}\right)$ is decreasing in $w_{1}$ and $\widetilde{w}_{1}(\delta)$ is decreasing in $\delta$. Finally, for $\delta \in[\widetilde{\widetilde{\delta}}, 1]$,

$$
\begin{gathered}
t_{1}=T_{12}\left(w_{12}(\delta), \delta\right)=\frac{1}{\delta}\left[\pi_{1}^{R}\left(w_{12}(\delta)\right)-\pi_{12}^{R}\left(w_{12}(\delta)\right)-\delta\left(\pi_{2 H}^{V I}-\pi_{12}^{R}\left(w_{12}(\delta)\right)\right)\right] \\
=\frac{1}{\delta}\left[\underline{q}\left(c_{2}-w_{12}(\delta)\right)\right]+\pi_{12}^{R}\left(w_{12}(\delta)\right)-\pi_{2 H}^{V I} .
\end{gathered}
$$

Differentiating $T_{12}\left(w_{12}(\delta), \delta\right)$ with respect to $\delta$ (notice that, by the envelope theorem $\frac{\partial \pi_{12}^{R}\left(w_{1}\right)}{\partial w_{1}}=$ $\left.-\widehat{q}_{1}\left(w_{1}\right)=-\left(q_{1}\left(w_{1}\right)-\underline{q}\right)\right)$ yields:

$$
\frac{d T_{12}\left(w_{12}(\delta), \delta\right)}{d \delta}=-\frac{q\left(c_{2}-w_{12}(\delta)\right)}{\delta^{2}}-\frac{\left[(1-\delta) \underline{q}+\delta q_{1}\left(w_{1}\right)\right]}{\delta} \frac{\partial w_{12}(\delta)}{\partial \delta}<0,
$$

where the first term is negative because $w_{12}(\delta)<c_{2}$ and the second term is negative because $\frac{\partial w_{12}(\delta)}{\partial \delta}>0$.

## Proof of Proposition 2:

The plan of the proof is as follows. In the first step, we show that for $\delta \in[0, \widetilde{\delta}]$, the gap in the manufacturer's profits from exclusion and accommodation is positive, hence $\delta^{V S}$ (if it exists) hinges on the higher ranges of $\delta$. In the second step, we show that for $\delta \in[\widetilde{\delta}, 1]$, the gap in the manufacturer's profits from exclusion and accommodation is decreasing in $\delta$. This implies that there is at most one threshold value of $\delta^{V S}$ such that the gap is positive if and only if $\delta \leq \delta^{V S}$. In the third step, we show that the gap in the manufacturer's profits from exclusion and accommodation becomes zero at some $\delta^{V S} \in[\widetilde{\delta}, 1]$, where $\delta^{V S}<\delta^{V I}$, and hence a unique threshold exists.

Step 1: Suppose first that $\delta \in[0, \widetilde{\delta}]$. The manufacturer sets $w_{1}=c_{1}$ and the retailer does not offer product 2. This is optimal for the manufacturer because the gap in the manufacturer's profits from
exclusion and accommodation is:

$$
\begin{equation*}
\frac{\Pi_{1}^{M}\left(c_{1}\right)}{1-\delta}-\Pi_{12}^{M}\left(c_{1}\right)=\frac{1}{1-\delta}\left[\pi_{1}^{V I}-\pi_{12}^{V I}-\delta\left(\pi_{2 H}^{V I}-\pi_{12}^{V I}\right)\right]>0, \tag{17}
\end{equation*}
$$

where $\Pi_{1}^{M}\left(c_{1}\right)$ is the first line of (6), $\Pi_{12}^{M}\left(c_{1}\right)$ is given by (9), and the inequality follows because the term in squared brackets is equivalent to (14), which is positive for $\delta<\delta^{V I}$ (and recall that $\widetilde{\delta}<\delta^{V I}$. Accordingly, it follows that if $\delta^{V S}$ exists, it has to be that: $\delta^{V S}>\widetilde{\delta}$.

Step 2: In this step we show that for $\delta \in[\widetilde{\delta}, 1]$, the gap in the manufacturer's profits from exclusion and accommodation is decreasing in $\delta$. When $\delta \in[\widetilde{\delta}, \widetilde{\widetilde{\delta}}]$, the gap in the manufacturer's profits from exclusion and accommodation is:

$$
\begin{equation*}
\frac{\Pi_{1}^{M}\left(\widetilde{w}_{1}(\delta)\right)}{1-\delta}-\Pi_{12}^{M}\left(c_{1}\right)=\frac{1}{1-\delta}\left[\pi_{1}^{V I}\left(\widetilde{w}_{1}(\delta)\right)-\pi_{12}^{V I}-\delta\left(\pi_{2 H}^{V I}-\pi_{12}^{V I}\right)\right], \tag{18}
\end{equation*}
$$

where $\Pi_{1}^{M}\left(\widetilde{w}_{1}(\delta)\right)$ is the second line in (6), evaluated at $w_{1}=\widetilde{w}_{1}(\delta)$, and $\Pi_{12}^{M}\left(c_{1}\right)$ is given by (9). Taking the derivative of (18) with respect to $\delta$ :

$$
-\frac{\pi_{2 H}^{V I}-\pi_{1}^{V I}\left(\widetilde{w}_{1}(\delta)\right)}{(1-\delta)^{2}}+\frac{1}{1-\delta} \frac{\partial \pi_{1}^{V I}\left(\widetilde{w}_{1}(\delta)\right)}{\partial w_{1}} \frac{d \widetilde{w}_{1}(\delta)}{d \delta}<0
$$

where the first term is negative because $\widetilde{w}_{1}(\delta)<c_{1}$ and $\pi_{2 H}^{V I}>\pi_{1}^{V I}>\pi_{1}^{V I}\left(\widetilde{w}_{1}(\delta)\right)$. The second term is negative because at the corner solution, $\frac{\partial \pi_{1}^{V I}\left(\widetilde{w}_{1}(\delta)\right)}{\partial w_{1}}$ is positive for $w_{1}<c_{1}$ while $\frac{d \widetilde{w}_{1}(\delta)}{d \delta}<0$.

When $\delta \in[\widetilde{\widetilde{\delta}}, 1]$, the gap in the manufacturer's profits from accommodation and exclusion is:

$$
\begin{align*}
\frac{\Pi_{1}^{M}\left(w_{12}(\delta)\right)}{1-\delta} & -\Pi_{12}^{M}\left(c_{1}\right)=\frac{1}{1-\delta}\left(\pi_{1}^{V I}\left(w_{12}(\delta)\right)-\pi_{2 H}^{V I}\right)  \tag{19}\\
& +\frac{1}{\delta}\left(\pi_{1}^{R}\left(w_{12}(\delta)\right)-\pi_{12}^{R}\left(w_{12}(\delta)\right)\right)-\Pi_{12}^{M}\left(c_{1}\right) .
\end{align*}
$$

The gap in (19) is strictly decreasing in $\delta$. This can be shown by taking the derivative of (19) with respect to $\delta$ and using the envelope theorem (notice that $\Pi_{1}^{M}\left(w_{12}(\delta)\right)$ is affected by $\delta$ both
directly and indirectly through $w_{12}(\delta)$ ), yielding:

$$
\begin{equation*}
\frac{\pi_{1}^{V I}\left(w_{12}(\delta)\right)-\pi_{2 H}^{V I}}{(1-\delta)^{2}}+\frac{\pi_{12}^{R}\left(w_{12}(\delta)\right)-\pi_{1}^{R}\left(w_{12}(\delta)\right)}{\delta^{2}} \tag{20}
\end{equation*}
$$

The first term in (20) is negative because $\pi_{1}^{V I}\left(w_{12}(\delta)\right)<\pi_{2 H}^{V I}$, while the second term is negative because $\pi_{12}^{R}\left(w_{12}(\delta)\right)<\pi_{1}^{R}\left(w_{12}(\delta)\right)$. We therefore have that $\Pi_{1}^{M}\left(w_{1}^{E}(\delta)\right)$ is decreasing in $\delta$ for $\delta \in[\widetilde{\delta}, 1]$, while $\Pi_{12}^{M}\left(c_{1}\right)$ is constant in $\delta$, implying that there is at most one threshold value of $\delta^{V S}$ such that the gap in the manufacturer's profits from exclusion and accommodation is positive if and only if $\delta \leq \delta^{V S}$.

Step 3: Now we turn to show that there exists a $\delta^{V S}<\delta^{V I}$ such that the gap in the manufacturer's profits from exclusion and accommodation is zero at $\delta^{V S}$. To this end, we show that this gap is negative at $\delta^{V I}$, whether $\delta^{V I} \in[\widetilde{\delta}, \widetilde{\widetilde{\delta}}]$ or $\delta^{V I} \in[\widetilde{\widetilde{\delta}}, 1]$. Because the gap is positive at $\widetilde{\delta}$, as shown in step 1 , and decreasing with $\delta$, as shown in step 2 , if it is negative at $\delta^{V I}$, it follows that it is zero for some $\delta \in\left(\widetilde{\delta}, \delta^{V I}\right)$. Suppose first that $\delta^{V I} \in[\widetilde{\delta}, \widetilde{\widetilde{\delta}}]$. Then, evaluated at $\delta=\delta^{V I}$, the term in the squared brackets in (18) is strictly negative, which follows from comparing it with the definition of $\delta^{V I}$ in (14) and because $\pi_{1}^{V I}\left(c_{1}\right)>\pi_{1}^{V I}\left(\widetilde{w}_{1}(\delta)\right)$. As $\pi_{1}^{V I}-\pi_{12}^{V I}-\delta\left(\pi_{2 H}^{V I}-\pi_{12}^{V I}\right)$ is always decreasing in $\delta$, it follows that if $\delta^{V I}<\widetilde{\widetilde{\delta}}$, there is a unique cutoff, $\delta^{V S},\left(\widetilde{\delta}<\delta^{V S}<\delta^{V I}\right)$, where $\delta^{V S}$ solves $\pi_{1}^{V I}\left(\widetilde{w}_{1}(\delta)\right)-\pi_{12}^{V I}=\delta\left(\pi_{2 H}^{V I}-\pi_{12}^{V I}\right)$, such that the manufacturer excludes (accommodates) product 2 if $\delta<\delta^{V S}\left(\delta>\delta^{V S}\right)$. Notice that in this case it has to be that $\delta^{V S}<\widetilde{\widetilde{\delta}}$.

Next, suppose that $\delta^{V I} \in[\widetilde{\widetilde{\delta}}, 1]$. The term in (17) places an upper bound on $\frac{\Pi_{1}^{M}\left(w_{12}(\delta)\right)}{1-\delta}-\Pi_{12}^{M}\left(c_{1}\right)$, because:

$$
\begin{aligned}
\frac{\Pi_{1}^{M}\left(w_{12}(\delta)\right)}{1-\delta}-\Pi_{12}^{M}\left(c_{1}\right) & =\frac{1}{1-\delta}\left[\pi_{1}^{M}\left(w_{12}(\delta)\right)+T_{12}\left(w_{12}(\delta), \delta\right)\right]-\Pi_{12}^{M}\left(c_{1}\right) \\
& <\frac{1}{1-\delta}\left[\pi_{1}^{M}\left(w_{12}(\delta)\right)+T_{2}\left(w_{12}(\delta), \delta\right)\right]-\Pi_{12}^{M}\left(c_{1}\right) \\
& =\frac{1}{1-\delta}\left[\pi_{1}^{V I}\left(w_{12}(\delta)\right)-\pi_{2}^{V I}-\delta\left(\pi_{2 H}^{V I}-\pi_{12}^{V I}\right)\right]-\Pi_{12}^{M}\left(c_{1}\right) \\
& <\frac{1}{1-\delta}\left[\pi_{1}^{V I}\left(c_{1}\right)-\pi_{2}^{V I}-\delta\left(\pi_{2 H}^{V I}-\pi_{12}^{V I}\right)\right]-\Pi_{12}^{M}\left(c_{1}\right) \\
& =\frac{1}{1-\delta}\left[\pi_{1}^{V I}-\pi_{12}^{V I}-\delta\left(\pi_{2 H}^{V I}-\pi_{12}^{V I}\right)\right],
\end{aligned}
$$

where the first inequality follows because $T_{12}\left(w_{12}(\delta), \delta\right)<T_{2}\left(w_{12}(\delta), \delta\right)$ and the last inequality follows because $w_{1}=c_{1}$ maximizes $\pi_{1}^{V I}(w)$. Because by the definition of $\delta^{V I}, \pi_{1}^{V I}-\pi_{12}^{V I}-\delta^{V I}\left(\pi_{2 H}^{V I}-\right.$
$\left.\pi_{12}^{V I}\right)=0$, we have that there is a $\delta^{V S}<\delta^{V I}$ such that $\frac{\Pi_{1}^{M}\left(w_{12}(\delta)\right)}{1-\delta}-\Pi_{12}^{M}\left(c_{1}\right)=0$ for $\delta=\delta^{V S}$. As the gap $\frac{\Pi_{1}^{M}\left(w_{12}(\delta)\right)}{1-\delta}-\Pi_{12}^{M}\left(c_{1}\right)$ is decreasing with $\delta$, the gap is positive iff $\delta<\delta^{V S}$. Notice that even when $\delta^{V I}>\widetilde{\widetilde{\delta}}$, it can still be that $\delta^{V S}<\widetilde{\widetilde{\delta}}<\delta^{V I}$.

## Proof of Proposition 3:

The result that for $\delta \in[0, \widetilde{\delta}]$, the manufacturer excludes product 2 without exclusive dealing follows directly from the proof of Proposition 2. Suppose now that $\delta>\widetilde{\delta}$. The gap in the manufacturer's profits when imposing exclusive dealing and accommodating product 2 is:

$$
\begin{align*}
\frac{\Pi_{1}^{M, E D}\left(c_{1}\right)}{1-\delta}-\Pi_{12}^{M}\left(c_{1}\right) & =\frac{\pi_{1}^{V I}-\pi_{2}^{V I}-\delta\left(\pi_{2 H}^{V I}-\pi_{2}^{V I}\right)}{1-\delta}-\left(\pi_{12}^{V I}-\pi_{2}^{V I}\right)  \tag{21}\\
& =\frac{\pi_{1}^{V I}-\pi_{12}^{V I}-\delta\left(\pi_{2 H}^{V I}-\pi_{12}^{V I}\right)}{1-\delta}
\end{align*}
$$

where the first equality follows from substituting (11) into $\Pi_{1}^{M, E D}\left(c_{1}\right)$ and (9) into $\Pi_{12}^{M}\left(c_{1}\right)$. Comparing the second line of (21) with the definition of $\delta^{V I}$ in (14) yields that $\frac{\Pi_{1}^{M, E D}\left(c_{1}\right)}{1-\delta}-\Pi_{12}^{M}\left(c_{1}\right)>0$ iff $\delta<\delta^{V I}$.

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    ${ }^{\dagger}$ Buchmann Faculty of Law, Tel Aviv University (email: gilod@tauex.tau.ac.il)
    ${ }^{\ddagger}$ Coller School of Management, Tel Aviv University (email: yehezkel@tauex.tau.ac.il)

[^1]:    ${ }^{1}$ Jabr and Rahman (2019) have documented surveys showing that $63 \%$ of customers consider it extremely or very important to read reviews online before buying an unfamiliar product, and that $93 \%$ of customers read online reviews before choosing what to buy.

[^2]:    ${ }^{2}$ We expect our qualitative results to extend to competing retailers who are sufficiently differentiated from each other and we comment on how the results change when competing retailers are homogeneous in subsection $\mathrm{VI}(\mathrm{iv})$. Also, the model applies, mutatis-mutandis, to an end-buyer of the product who is not necessarily a retailer. Some of our policy implications, though, concern a reduction in the price the retailer charges end-consumers below the monopoly price.

[^3]:    ${ }^{3}$ Note that such behavior qualifies as 'predatory pricing' according to commonly used economic definitions, such as that of Bolton et al [1999], who define it as '... [A] price reduction that is profitable only because of ... its exclusionary ... effects.' To be sure, the 'persistent predatory pricing' we identify, alongside its exclusionary effect, also has a pro-competitive effect, since it reduces retail prices below the monopoly price.

[^4]:    ${ }^{4}$ Carlton and Waldman (2002), who focus on tying in a finite game, remark that in dynamic technological industries, in which products become obsolete, products' lifetimes are limited.

[^5]:    ${ }^{5}$ The exclusionary mechanism that our paper identifies holds when there is a monopolistic retailer (and trivially follows to the case where competing retailers are highly differentiated). Yet, this mechanism may not hold in the case of competition between homogeneous retailers. We comment on the case of competing retailers in subsection VI(iv).
    ${ }^{6}$ As we explain in subsection $\mathrm{VI}(\mathrm{i})$ and show in detail in Online Appendix A, our results qualitatively extend to a model in which product 2 is sold by a strategic player. In particular, we show that such a strategic player always offers a wholesale price equal to its marginal costs and consequently the manufacturer offers the same exclusionary contract as in our main model. Online Appendix A is available at: https://www.tau.ac.il/~yehezkel/Appendix\%20A_strategic\%20M2.pdf

[^6]:    ${ }^{7}$ The assumption that $\underline{q}<q_{2}^{V I}$ is not necessary for our main results. In Section III, after Corollary 1, we discuss the implications of larger levels of $\underline{q}$.
    ${ }^{8}$ If product 2's improvement does not exceed the profitability of product 1, i.e., $\pi_{2 H}^{V I}>\pi_{2}^{V I}$ but $\pi_{2 H}^{V I}<\pi_{1}^{V I}$, then under vertical integration it is never profitable to sell product 2. Under vertical separation, the retailer might find it optimal to sell product 2 for one period to improve it just in order to enhance the retailer's outside option in future periods. Yet, in equilibrium, the retailer will not sell the improved product 2 in future periods.

[^7]:    ${ }^{9}$ Because of the dynamic nature of the game, there are potentially multiple equilibria. In particular, if the manufacturer expects that the retailer accommodates product 2 regardless of the manufacturer's contract, the manufacturer will offer the accommodation contract that we identify in the second part of this section, and the retailer will indeed accept the offer and accommodate product 2 . Such an equilibrium, however, is not sequentially rational, since the manufacturer can offer an exclusionary contract that the retailer accepts.

[^8]:    ${ }^{10}$ Note that $q_{1}\left(w_{1}\right)$ solves $p^{\prime}\left(q_{1}\right) q_{1}+p\left(q_{1}\right)-w_{1}=0$ while $\widehat{q}_{1}\left(w_{1}\right)$ solves $p^{\prime}\left(q_{1}+\underline{q}\right)\left(q_{1}+\underline{q}\right)+p\left(q_{1}+\underline{q}\right)-w_{1}=0$. Hence $\widehat{q}_{1}\left(w_{1}\right)=q_{1}\left(w_{1}\right)-\underline{q}$. As noted, the crucial feature of $\widehat{q}_{1}\left(w_{1}\right)$ for our results is that $\widehat{q}_{1}\left(w_{1}\right)<q_{1}\left(w_{1}\right)$.

[^9]:    ${ }^{11}$ See Section VI(ii) and Online Appendix D.

[^10]:    ${ }^{12}$ As we show in Proposition 2, the manufacturer's exclusionary profits are decreasing in $\delta$, so it accommodates product 2 for $\delta$ strictly below $\delta^{V I}<1$. As demonstrated in the linear demand example below, this threshold of $\delta$ may lie either in the high range of $\delta((\delta \in[\widetilde{\widetilde{\delta}}, 1))$ or the intermediate range $((\delta \in[\widetilde{\delta}, \widetilde{\widetilde{\delta}}]))$, depending on market circumstances.
    ${ }^{13}$ See subsection VI(ii) and Online Appendix D.

[^11]:    ${ }^{14}$ See subsection VI(ii) and Online Appendix D.

[^12]:    ${ }^{15}$ The proof of the first part follows because, by equation (15) in the appendix, for any $w_{1} \leq c_{1}$, if $\pi_{2 H}^{V I}=\pi_{2}^{V I}$ then $w_{1}<\widetilde{w}_{1}(\delta)$ such that selling 2 binds. The second part is immediate.

[^13]:    ${ }^{16}$ See Online Appendix C at: https://www.tau.ac.il/~yehezkel/Appendix_C all_or_nothing_clause.pdf

[^14]:    ${ }^{17}$ In particular, $\widetilde{\delta}^{V S}<\delta^{V S}$ because when $\delta>\widetilde{\delta}$, the manufacturer is constrained by $t_{1} \leq T_{12}\left(w_{1}, \delta\right)$ whether $w_{1}=w_{1}^{E}(\delta)$ or $w_{1}=c_{1}$, but by revealed preference, the former case provides higher exclusionary profits. Moreover, $\widetilde{\delta}<\widetilde{\delta}^{V S}$ because evaluated at $\delta=\widetilde{\delta}$, exclusion at $w_{1}=c_{1}$ provides the manufacturer with strictly higher profits than accommodation (see inequality (17) in the appendix).
    ${ }^{18}$ See FTC, in The Matter of Broadcom Inc., July 2, 2021, available at https://www.ftc.gov/enforcement/cases-proceedings/181-0205/broadcom-incorporated-matter. The EU Commission reached a consent decree blocking similar practices by Broadcom in October 2020. See https://ec.europa.eu/competition/elojade/isef/case_details.cfm?proc_code=1_40608.

[^15]:    ${ }^{19}$ ZF Meritor v. Eaton Corp (3d. Cir.) 696 F.3d 254 (2012).
    ${ }^{20}$ (11th Cir.) 783 F.3d 814 (2015).
    ${ }^{21}$ Other examples of explicit exclusive dealing in dynamic settings similar to ours exist with respect to a dominant hospital's exclusive dealing with insurers where access to the insurers would have enabled the new hospital to improve (e.g., Methodist Health Servs. Corp. v. OSF Healthcare Sys. (United States District Court for the Central District of Illinois, Peoria Division) 2016 U.S. Dist. LEXIS 136478, at 293-294; Rome Ambulatory Surgical Ctr. v. Rome Mem. Hosp. (United States District Court for the Northern District of New York) 349 F. Supp. 2d 389 (2004).), or

[^16]:    ${ }^{24}$ For a more comprehensive comparison of all three antitrust regimes, see Online Appendix B, available at https://www.tau.ac.il/~yehezkel/Appendix\%20B _welfare.pdf.

[^17]:    ${ }^{25}$ Online Appendix B shows how a ban on persistent predatory pricing (regime III) is socially harmful due to its price-increasing effect, but may cause either over-accommodation or over-exclusion of product 2, depending on market circumstances.
    ${ }^{26}$ See, e.g., Brooke Grp. Ltd. v. Brown \& Williamson Tobacco Corp., 509 U.S. 209 (1993); ZF Meritor v. Eaton Corp (3d. Cir.) 696 F.3d 254 (2012).

[^18]:    ${ }^{27}$ See, e.g., Bork (1993) at p. 304; Knight and Windell (2001); Moore and Wright (2015); McWane, Inc. v. FTC (11th Cir.) 783 F.3d 814 (2015) (" ... courts often take a permissive view of [exclusive dealing] contracts on the grounds that firms compete for exclusivity by offering procompetitive inducements (e.g., lower prices, better service).".
    ${ }^{28}$ See Maxon Hyundai Mazda v. Carfax, Inc., 2016 U.S. Dist. LEXIS 171418.

[^19]:    ${ }^{29}$ In Online Appendix A we show that if the manufacturer of product 1 merges with the retailer, the vertically integrated outcome is replicated also when product 2 is supplied by a strategic player.

[^20]:    ${ }^{30}$ See Online Appendix D at: https://www.tau.ac.il/~yehezkel/Appendix_D_finite_game.pdf

[^21]:    ${ }^{31}$ In the context of exclusive agreements, the US court of appeals in FTC v. Qualcomm Inc. (9th Cir.) 969 F. 3 d 974,1003 (2020) held that "[w]e decline to ascribe antitrust liability in these dynamic and rapidly changing technology markets without clearer proof of anticompetitive effect". The same approach was echoed by the US district court in Epic Games v. Apple Inc., 2020 U.S. Dist. LEXIS 188668. Similarly, when approving the vertical merger between AT\&T and Time Warner, the US district court (affirmed by the court of appeals) was influenced by the dynamic nature of their industries, saying that it is required "to examine this case with an eye toward the 'structure, history, and probable future' of this fast-changing industry." United States v. AT\&T Inc. (District of Columbia) 310 F. Supp. 3d 161 (2018), at footnote 6.

[^22]:    ${ }^{32}$ This, in turn, can raise further antitrust implications regarding exclusive dealing or slotting allowances adopted by the manufacturer of product 2 attempting to monopolize the market with its improvable product. Some jurisdictions, for example, ban negative fees altogether, particularly in the food sector, such as the Israeli Food Law.

[^23]:    ${ }^{33}$ With Bertrand retail competition, this would not cause double-marginalization.

[^24]:    ${ }^{34}$ We should note that the manufacturer's optimal strategy may involve $w_{1}^{E}(\delta)=\widetilde{w}_{1}(\widetilde{\widetilde{\delta}})<0$. If a negative wholesale price is impossible to implement, then there are two cutoffs of $\delta, \delta^{\prime}<\widetilde{\widetilde{\delta}}<\delta^{\prime \prime}$ such that the manufacturer sets $w_{1}(\delta)=0$ for $\delta \in\left[\delta^{\prime}, \delta^{\prime \prime}\right]$. This will not qualitatively change our results.

