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Modular liability rules*

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Abstract

Recent work has discussed and formalized the conditions under which liability rules are superior to property rules in a world with "one-sided" incomplete information. These studies demonstrated how liability rules achieve higher social welfare by harnessing one party's private information about its own valuation to the process of optimally allocating the entitlement between the parties. This article introduces a new family of liability rules hitherto neglected by courts and legal scholars. A regime of modular liability rules is one in which the court applies legal rules for which the traditional liability rules are building blocks; these rules harness both parties' private information. Whereas in an efficient liability rule 2 (for example) a polluter is granted a call-option to purchase the right to the air (so to speak) with an exercise price that equals the resident's harm, in modular liability rule 6 + 5 (for example) a pair of options, rather than a single one, is allocated; the resident gets a put option to force a transfer of the entitlement (as under rule 6), but the polluter has a consecutive put option to sell the entitlement back to the resident, if he wishes (as under rule 5). Interestingly, the maximizing joint welfare exercise-price equals, in general and for uniform distributions, an amount that is the average of one party's maximum estimated valuation and the other party's minimum estimated valuation. Two paradigmatic worlds of two-sided incomplete information are studied: a symmetric world, in which the court's best estimate of the parties' private valuations is that they are identically distributed, and an asymmetric world. In the symmetric world, modular liability rules are in some respects more efficient and more fair (exact definitions are discussed in the article) than the conventional liability rules. In the two-sided and asymmetric world, modular liability rules yield higher joint payoffs than regular liability rules if, and only if, the difference between the parties' means is larger than the difference between their amount of private information (represented by the distribution's support). A practical way to implement these insights in real life situations is offered.

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1. Introduction

The construction of *socially optimal legal rules* for solving disputes where the core conflict is the allocation of entitlement in an environment of incomplete information is challenging. One common instance of this challenge is the difficulty in resolving nuisance disputes, in which the parties are in conflict over the "right to the air." The term *Legal rules* refers to the courts' optional ways of allocating entitlements between parties accompanied by money transfers between them. *Socially optimal legal rules* refers to the social goal of maximizing total welfare by allocating the entitlement to the highest (ex-post) valuer and flexibly employing fairness or distributional considerations.[1] *Incomplete information* refers to an environment where parties' valuation of the entitlement is private and therefore unknown to the social planner.

Ever since Calabresi and Melamed introduced the celebrated framework of the four legal rules that courts can use in solving disputes, the issue of finding the socially optimal legal rule has been a subject of academic interest.[2] In particular, scholars have debated the relative advantages and disadvantages of property rules in comparison with liability rules. Recent work has discussed and formalized Calabresi and Melamed's insight for the conditions under which liability rules are superior to property rules in a world with "one-sided" incomplete information.[3] These studies demonstrated how liability rules achieve higher social welfare by harnessing *one* party's private information about its own valuation to the process of optimally allocating the entitlement between the parties.[4]

This article extends this literature in two major ways. First, it introduces a new family of liability rules hitherto neglected by courts and legal scholars. A regime of *modular liability rules* (MLRs) combines the singular liability rules. In MLRs the court applies legal rules for which the regular liability rules are building blocks. In this regime, a *pair* of options, rather than a single one, is allocated. MLRs harness *both* parties' private information to the process of allocating the entitlement in dispute. Thus, this article attempts to systematically analyze, and this is its second contribution to the contemporary literature, a world of "two-sided" incomplete information—a generalized world in which *both parties'* exact valuations of the entitlement are unknown to the court or to the other party. Two paradigmatic worlds of this type are studied: a *symmetric world*, in which the court's best estimate of the parties' private valuations is that they are identically distributed, and an *asymmetric world*, in which the court knows that the parties' distributions of valuation differ.[5]

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[1] Administrative costs and costs of bearing risks are neglected at this stage.

[2] Calabresi and Melamed Property Rules, Liability Rules, and Inalienability: One View of the Cathedral, 85 HARV. L. REV. 1089 (1972) (hereinafter the Cathedral).

[3] Kaplow and Shavell, Property Rules versus Liability Rules: An Economic Analysis, 109 HARV. L. REV. 715 (1996) (hereinafter Economic Analysis); I. Ayres, Monsanto Lecture, Protecting Property with Puts, 32 VAL. U. L. REV. 793 (1998) (hereinafter Protecting with Puts).

[4] For example, liability rule 2, which grants the polluter the opportunity to pay damages and continue polluting, provides the polluter with incentives to "purchase" the right to the air, only when his private valuation is higher than the court's determined compensation. Put differently, the polluter is granted a call-option to purchase the right to the air with an exercise price that equals the resident's harm, which he will exercise if, and only if, his private valuation is higher than the exercise price.

[5] In a recent discussion paper, which was circulated after this paper was written, Ayres and Goldbart elegantly explore the same family of rules analyzed in this paper. *See* Ayres and Goldbart, *Optimal Delegation and Decoupling*

The remainder of this article is organized as follows: Section 2 presents a theoretical viewpoint based on an option analysis that serves as a framework for an analysis of the conventional legal rules. In Section 3, a world of two-sided and *symmetric* incomplete information is discussed. Section 4 introduces the new *modular liability rule*. Sections 5–7 focus on a world of two-sided and *asymmetric* incomplete information. In this world, MLRs are, when holding the dispersion (or variance) of the parties' private valuations equal, *more* efficient than regular liability rules. In contrast, when holding the *means* of the parties' private valuation equal, MLRs are *less* efficient than regular liability rules. More generally, for uniform distributions, when the difference between the parties' means is larger than the difference between the parties' private information (all in absolute values), MLRs dominate. A somewhat surprising corollary of the analysis presented in this article is that, unlike conventional liability rules in which the optimal damages set by the court equals the aggrieved party's expected harm, in MLRs the optimal damages set by the court, are, in general, equal to an amount that is the average of one party's maximum estimated valuation and the other party's minimum estimated valuation.[6] In Section 8 unifying insights and a practical way to implement these insights in real life situations are offered.

2. Property rules versus liability rules—the literature

Traditionally, the literature has dealt with six major rules (labeled rules 1–6), four of which were presented by Calabresi and Melamed. As applied to the now canonical stylized nuisance dispute, the four rules are: property rule 1, in which the court issues an injunction against a polluter; liability rule 2, in which the court finds a nuisance but permits continued pollution if the polluter pays damages; property rule 3, in which the court finds no nuisance and permits the polluter to continue without paying damages; and lastly, liability rule 4, in which the court permits a polluter to continue operating unless the resident pays the polluter damages to constrain further pollution.[7]

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in the Design of Liability Rules (on file with the author). Their paper also provides a formal model and develops similar graphical tools. Ayres and Goldbart's paper offers important insights regarding the effects of MLRs (which they call a defendant (or plaintiff)-joint-veto-rule). This paper differs in several respects. First, this paper provides deeper analysis of some of the results reported by Ayres and Goldbart. In particular, it has a stronger and more detailed analysis of whether various rules violate litigant participation constraints. Second, it also provides an original and detailed analysis of the way to derive optimal damages. Third, this paper provides different and original interpretations of the results and their underlying intuitions. Fourth, this paper provides formal proofs to some of Ayres and Goldbart's assertions. Fifth, this paper includes a simple way to implement the theoretical insights developed in the paper.

[6] For some distributions, the optimal damages (for both regular liability rules and MLRs) equal either the maximum or the minimum estimated valuation of one party's valuation. *See infra* Sections 4–7.

[7] Indeed, this rule was implemented by a court in the famous case, *Spur Industries v. Del Webb Development Co.*, 494 P.2d 700 (Ariz. 1972). For criticism of rule 4 see Richard Epstein, *A Clear View of The Cathedral: The Dominance of Property Rules*. 106 Yale L.J. 2091, 2103–05 (1997) (hereinafter *Dominance of Property Rules*) ("Apart from that flirtation [Spur Industries], Calabresi and Melamed's fourth alternative has largely been ignored in nuisance cases, and for good reason: it utterly subverts the nature of property rights"). Epstein concludes by stating that "rule 4 should disappear from the face of the earth." *Id*. at 2120.

For the past decade, Calabresi and Melamed's liability rules 2 and 4 have been interpreted as types of *call options*.[8] A call option is the choice of the option-holder to force a non-consensual purchase on the entitlement-holder for a pre-determined price. Under rule 2, the polluter effectively has a call option to buy the resident's entitlement to clean air. Similarly, rule 4 involves a call option, but one that is held by the resident. Rules 5 and 6, which evolved in the 1990s, also involve the granting of options, but instead of call options, they grant *put options*. A put option is the choice whether to keep the entitlement or to force a non-consensual *purchase* by the non-holder.[9] Table 1 summarizes:

Table 1
A comparison of rules 1–6

Rule	Legal implication	Holder of the entitlement	Holder of the option
1	Injunction against the polluter—no more harm is caused to the resident	Resident	
2	Polluter decides whether to compensate the resident or to shut off his plant	Resident	Polluter (call)
6	Resident decides whether to get paid and suffer from the pollution or to shut off the polluter's plant	Resident	Resident (put)
3	No nuisance—the polluter can continue to pollute without paying damages	Polluter	
4	Resident decides whether to compensate the polluter for shutting off his plant or to suffer from pollution	Polluter	Resident (call)
5	Polluter decides whether to get paid for stop polluting or to continue polluting	Polluter	Polluter (put)

[8] M. Morris, *The Structure of Entitlements*, 78 Cornell L. Rev. 822 (1993); J.E. Krier and S.J. Schwab *Property Rules and Liability Rules: The Cathedral in Another Light*, 70 N.Y.U L. Rev 440 (1995) (hereinafter *Another Light*); I. Ayres and E. Talley, *Solomonic Bargaining: Dividing a Legal Entitlement to Facilitate Coasean Trade*, 104 Yale L.J. 1027 (1995); Kaplow and Shavel *Economic Analysis supra* note 3; I. Ayres and J.M. Balkin, *Legal Entitlements as Auctions: Property Rules, Liability Rules, and Beyond*, 106 Yale L.J. 703 (1996) (hereinafter *Legal entitlements as Auctions*); Epstein *The Dominance of Property Rules, supra* note 3; Saul Levmore, *Unifying Remedies: Property Rules, Liability Rules, and Startling Rules*, 106 Yale L.J. 2149 (1997); Carol M. Rose, *The Shadow of the Cathedral*, 106 Yale L.J. 2175 (1997); Epstein, *Commentary: Protecting Property Rights with Legal Remedies: A Common Sense Reply To Professor Ayres*, 32 Val. U. L. Rev. 833 (hereinafter *A Common Sense Reply*); Ayres, *Protecting with puts supra* note 3.

[9] Unlike rules 2 and 4, which grant the (call) option to the party who does not hold the entitlement, rules 5 and 6 grant the (put) option to the party who does hold the entitlement. Specifically, in rule 5 (originally suggested by Krier and Schwab), the court allows a polluter to choose between two courses of action. The polluter can keep the entitlement and continue polluting (without paying any damages), or the polluter can, by exercising his put option, stop polluting, in which case the polluter is entitled to damages from the resident. Analogously, in rule 6 (cogently developed by Ayres), the court allows the resident to choose between an injunction, which would terminate the pollution without her paying any damages to the polluter, and waiving her injunctive rights, in which case she is entitled to damages from the polluter in return. *Protecting with puts supra* note 3. Rule 6 was however first suggested (and rejected as not feasible) by Krier and Schwab *Another Light supra* note 8 at note 92. As Ayres mentioned, in the classic 1895 encroachment case Pile v. Pedrick (31 A. 646 (Pa. 1895)), Pedrick mistakenly built a factory wall with a foundation that encroached 1 and 3/8 inches onto Pile's land. The court offered Pile an option: either damages for the permanent trespass or an order to remove the wall. Pile preferred to receive the damages. *Id*.

As Table 1 reveals, there is symmetry among the six rules. This article starts by focusing on the first group, property rule 1, liability rules 2 and 6, each of which assumes that the entitlement is initially held by the resident; later in the article the second group (and some other rules) will be incorporated into the analysis.

3. Property rules versus liability rules in a world of two-sided and symmetric incomplete information

3.1. Framework for comparing liability rules and property rules

The following simplifying assumptions are made: a single risk neutral polluter and a single risk neutral nearby resident are disputing the entitlement to the air between them.[10] The polluter has only one possible level of operation. There are no more cost-justified precautions that either of the parties can take in order to reduce the harm caused by the pollution. As a result, the court is assumed to face a problem of allocating a single unit of an indivisible good.

It is further assumed that the court's estimate is that parties' private valuations are uniformly and independently distributed along the intervals $[\mu_\mathrm{r} - r, \mu_\mathrm{r} + r]$ and $[\mu_\mathrm{p} - p, \mu_\mathrm{p} + p]$. Where are the polluter's and resident's mean valuations, and p and r represent half of their distribution dispersion, respectively. The higher p or r the more private information parties have vis-à-vis the court.[11] For simplicity it is first assumed that both parties' distributions lies in the interval [0,100], so that $\mu_\mathrm{p} = \mu_\mathrm{p} = p = r$.

As the court's best estimate is that on average the polluter's and the resident's valuations are 50, the court can not a priori know who the highest valuer is. Note that at this stage it is assumed that the court's incomplete information about the parties' valuations is *symmetric*; namely, in both cases the court assumes the *same* uniform distribution for the parties' valuations.

It is further assumed that parties are barred from negotiation before and after the court's decision. This assumption makes the ex-post efficiency inquiry non-trivial. Finally, it is assumed that all court estimates are immune from biased errors. The next sections compare the allocative efficiency of applying the different rules to the above-described dispute.

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[10] Both parties are assumed to have additively separable and linear utility both in money and the entitlement (quasi-linear preferences). Without loss of generality the parties' initial endowment of the numeraire ("money") can be normalized to zero. These assumptions allow us to ignore income-effects.

[11] Underlying the two-sided incomplete information assumption is the fact that, on the one hand, the resident's house may be a unique property with no market value, or it may be a family estate for which the resident has intense personal feelings of attachment. The polluter, on the other hand, may have just constructed a new plant to enter a new market and has not yet generated revenue so not enough information could be presented before the court for it to construct a concrete distribution function. As each party's private valuation is based on a different source, their valuations are independent of each other.

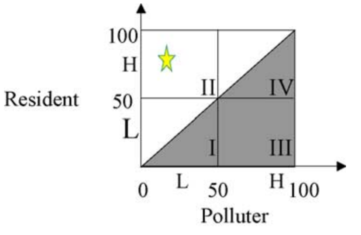

Chart 1. The first-best allocation of entitlement.

3.2. First-best and second-best allocations

The socially first-best solution is the maximum attainable joint welfare in a regime of complete information. This solution is achieved when the entitlement is allocated to the highest (ex-post) valuer (Chart 1 presents this graphically). This allocation will serve as a benchmark against which the results of the various rules under consideration will be compared.

In Chart 1, the horizontal axis represents the polluter's possible valuations and the vertical axis represents the resident's. The diagonal represents an indifference line, dividing the square into two possible allocations. The white triangle is where the first-best solution dictates giving the resident the entitlement because his valuation is higher then the polluter's. Conversely, the gray triangle is where the polluter, as the highest valuer, should receive the entitlement to accomplish the first-best solution. In the appendix it is shown that the first-best solution yields an ex-ante joint payoff of 66.67. Unfortunately, under our assumptions, there is no *realistic* rule[12] the social planner can apply and achieve the first-best solution.[13]

[12] A realistic rule means a rule that satisfies the following constraints: First, it presupposes that one of the parties has been preassigned a right to the entitlement; for now we will assume that this party is the resident. Samuelson showed that in the absence of a preassignment of the right to the entitlement, parties can achieve first-best by a split-the-difference bidding mechanism. Samuelson's bidding mechanism induces strategic behavior by the parties and violates other constraints as well. W. Samuelson, *A comment on the Coase theorem*, in Roth, Game-Theoretic Models of Bargaining 321, 330 (1985). Second, the mechanism induces no strategic behavior by the parties. That is, the social planner should design a mechanism where it is a dominant strategy for parties, acting in their self-interest, to act compatible to their *true* preferences and not to behave strategically. Ayres and Balkin designed a mechanism that at the limit approached the first-best solution. However, their mechanism (which mimicked an ascending auction for the entitlement) induced strategic behavior by both parties and thus violated this constraint. In addition, it was not as simple to implement as the ones we will consider below. *See Legal Entitlement as Auctions supra* note 8. Third, it should be individually rational for each party to follow, ex-post, the rule. Thus, the polluter should not be forced to pay damages for future pollution after his plant shuts down.

[13] This is a corollary from the celebrated impossibility theorem by Myerson and Satterthwaite which states that when parties' private valuations are independently distributed with positive probability density over some intervals and, if gains from trade are possible but are not certain (the interior of the parties' intervals have a non empty intersection), then no Bayesian incentive compatible (BIC), interim individually rational (IIR) trading mechanism can be ex-post efficient. *See* Myerson and Satterthwaite, *Efficient Mechanism For Bilateral Trading*, Journal of Economic Theory 29, 265 (1983). Williams has shown the exact relationship between the impossibility result and the underlying assumptions. For some forms of intersection of the distributions and when there is sufficiently large difference between the number of the parties on each side the impossibility disappears. S.R.Williams, *A characterization of efficient Bayesian incentive compatible mechanisms*, Econ. Theory 14, 155, 169 (1999).

3.3. Allocation in the shadow of property rule 1

Under property rule 1, because there is no post-trial transfer possible, the resident is always expected to retain the entitlement, which has an average value of 50. The polluter will never pollute and has a welfare of 0. Thus, the ex-ante joint welfare of the allocation equals the resident's expected valuation of 50.

Lemma 1. *In the symmetric world, allocation in the shadow of property rule 1 yields the following results*: (a) *the maximum joint ex-ante payoff equals the resident's expected payoff*, (b) *the resident's expected payoff equals her expected valuation, and* (c) *the polluter's expected payoff is zero*.

Remarks. (a) Under our assumptions, the resident's expected payoff is 50 and the polluter's expected payoff is zero; $E[V_{\mathrm{r}}]=50$ and $E[V_{\mathrm{p}}]=0$, respectively. (b) Under our assumptions, rule 1 yields the minimum ex-ante joint payoff.

3.4. Allocation in the shadow of liability rule 2

When applying liability rule 2, courts should first set the exercise price. Efficiency dictates that it should be set to maximize parties' joint payoff. As Kaplow and Shavell proved, the optimal exercise price equals the resident's *expected* valuation.

Given this design, the polluter will exercise his call option only when his subjective valuation is higher than the exercise price of 50, which is expected to occur 50% of the time. Exercising the call option by a "high" polluter yields him, on average, a profit of 25 (the polluter's valuation in this case is between 50 and 100 – on average 75 – and the damages he pays are 50). When the polluter's subjective value of the entitlement is lower than 50, he prefers to shut down his plant, in which case his valuation is, on average, 25. Shutting down is expected to occur 50% of the time, in which case the polluter's final welfare is zero. Thus, the polluter's expected welfare is 12.5. Fig. 1 shows the parties' possible actions (Chart 2):

As Fig. 1 shows, joint welfare is, on average, 62.5. Chart 3 demonstrates that when the polluter is "high" (cells III and IV) he will hold the entitlement.

The following lemma can be made:

D'aspremont and Gerard-Varet have shown that it is possible to achieve ex-post efficiency with a mechanism that is strategy-proof and budget balanced, though a party, who knows her valuation but not that of the other party, will do worse under the D'aspremont and Gerard-Varet mechanism than if she did not participate at all. D'aspremont and Gerard-Varet, *Incentive and incomplete information*, J. Pub. Econ. 11, 35 (1979). Fourth, all the money paid by one party reaches the other. In our model it means that the compensation paid to the payee was collected from, and only from, the payor. Meyerson and Satterthwaite showed that if the government is willing to subsidize the mechanism (and violate the budget-balance constraint) then there *is* a mechanism that is strategy-proof, rational and ex-post efficient. Myerson and Satterthwaite *Id*. Overall, the rule must be reasonably implementable in real-life settings. Under these constraints, the maximum attainable joint payoff is 62.5. Hagerty and Rogerson have shown that posted-price mechanisms (mechanisms where a price is posted in advance and transfer occurs if, and only if, both parties agree to the transfer) are essentially the only mechanisms such that each party has a dominant strategy. As will be shown below, modular liability rules are, in essence, posted-price mechanisms, and they yield a joint payoff of 62.5. Thus, this is the second-best joint payoff that satisfies the above constraints. Hagerty amd Rogerson, *Robust Trading Mechanisms*, Journal of Econ. Theory 42, 94 (1987).

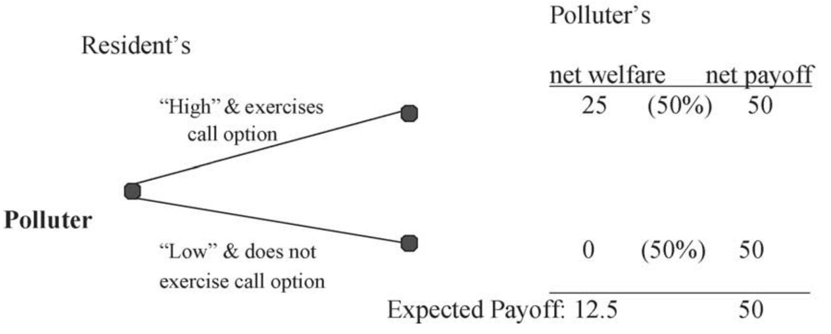

Fig. 1. Liability rule 2, the polluter has a call option.

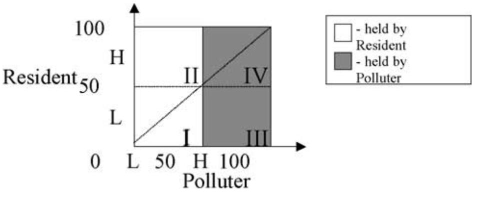

Chart 2. Liability rule 2.

Lemma 2. *In the symmetric world, allocation in the shadow of liability rule* 2 *yields the following results*: (a) *the optimal exercise price* (D^*) *equals the resident's expected value* (Kapow & Shavell, 1996), (b) *when setting the damages to equal* D^*, *the joint ex-ante payoff*, J^*, *reaches the second-best solution*, (c) *the resident's expected payoff equals her expected valuation, and* (d) *the polluter's expected payoff equals one quarter of p*.

Remarks. (a) Proofs of parts (b)–(d) are given in the appendix.
(b) Under our assumptions $D^* = 50$, $E[V_r] = 50$, $E[V_p] = 12.5$ and $J^* = 62.5$.
(c) Intuition. Part (b): Rule 2 does not reach the first-best result because when the polluter exercises his option, one cannot know whether the resident's valuation is even higher, in which case the entitlement did not end up with the highest valuer.[14] As Chart 3 shows, rule 2 causes both efficient transfers and non-transfer[15] as well as inefficient transfers and non-transfers.[16]

Part (c): The polluter will take whenever he is high, in which case the resident receives damages equal to her expected valuation. Otherwise, the resident holds the entitlement. Regardless, the resident receives her ex-ante expected valuation.

Part (d): In a symmetric world the polluter will take, 50% of the time. If the polluter is high, his expected valuation equals the average between the resident's mean and his maximum

[14] The result relies on the independence assumption.

[15] Cell III, in which the polluter is high and the resident is low and cell II, in which the resident is high and the polluter is low, respectively.

[16] The northwest triangle in cell IV, and the southeast triangle in cell I, respectively.

estimated valuation, $\mu_{\mathrm{p}} + p$. Thus, given that he pays damages equal to the resident's mean value, his payoff is as stated in Lemma 2.

(d) Distribution. Polluter: Under rule 2 he receives an option to force a transfer that he did not have under rule 1, and his expected payoff therefore increases from 0 to 12.5. Resident: Within the ex-ante perspective the resident is not worse off under rule 2 than under rule 1; in both cases she has an ex-ante payoff of 50. In the ex-post perspective however, there is 25% chance that the resident will find herself with a negative payoff, as she received 50 for an entitlement she may value at more than 50.
(e) Division of the surplus. Switching from rule 1 to rule 2 allocates the entire expected surplus created from this move to the polluter.
(f) Efficiency criterion. Whereas, ex-ante, rule 2 Pareto dominates rule 1, it does not do so ex-post.[17]
(g) Risk aversion. A risk averse resident may prefer property rule 1 to liability rule 2, despite an equal ex-ante expected payoff of 50. The reason is that under rule 1 she gets the expected payoff with no variance, whereas under rule 2 she has an expected payoff of 50 with 50%, a payoff of 75 with 25% and a payoff of (−25) with 25%.

3.5. Allocation in the shadow of liability rule 6

When applying liability rule 6 (a put option to the resident) courts should first set the exercise price (the damages the polluter should pay to the resident). As Ayres argued, the rule 6 optimal exercise price equals the polluter's *expected* valuation.

In this design, the resident will exercise her put option 50% of the time when her subjective valuation is lower than the exercise price of 50, yielding her a payoff of 50. When the resident is high she would prefer not to exercise her put option, in which case her valuation is, on average, 75. The expected joint welfare is 62.5.[18] The polluter, on the other hand, has 0 expected payoff. Half of the time, when the resident does exercise her put option the polluter pays 50 and gets the entitlement, which is also worth to him, on average, 50. The other half of the time, when the resident does not exercise her put option, the polluter remains with a payoff of 0. Fig. 2 represents this case:

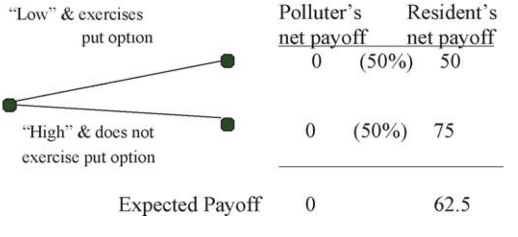

Fig. 2. Liability rule 6, the resident has a put option.

[17] To see it, observe that the change of colors in cell IV from white (in Chart 2) to gray (in Chart 3), from an allocative efficiency point of view, has no significance because there is no difference between who holds it. The interpretation of the change of colors in cell IV is that the polluter's gain equals the resident's loss.

[18] $1/2 \times (50) + 1/2 \times (75) = 62.5$.

Chart 3 shows that whenever the resident is low, she manages to force a transfer on the polluter:

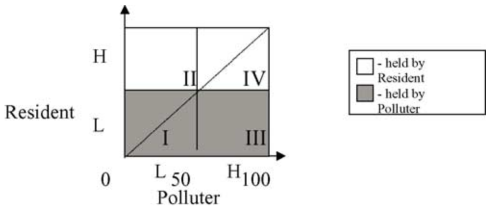

Chart 3. Liability rule 6.

The following lemma can be made:

Lemma 3. *In the symmetric world, allocation in the shadow of liability rule 6 yields the following results*: (a) *the optimal exercise price* (D^*) *equals polluter's expected value* (Ayres, 1998), (b) *when setting the damages to equal* D^*, *the joint ex-ante payoff*, J^*, *reaches the second-best solution*, (c) *the resident's expected payoff equals her expected valuation plus* 1/4 *of* r, (d) *the polluter's expected payoff equals zero*.

Remarks. (a) Proofs of parts b to d are given in the appendix.
(b) Under our assumptions $D^* = 50$, $E[V_r] = 62.5$ and $J^* = 62.5$.
(c) Intuition. This discussion is omitted because it is symmetric with the discussion of rule 2.

(d) Comparison with rule 2. Both rules yield the ex-ante second-best solution of 62.5, because in both there are too many transfers and too many non-transfers of the entitlement. Both rules accomplish this by using the option-holder's private information, allocating the surplus to the option holder and switching to them from property rule 1 is within the ex-ante, but not within the ex-post perspectives, a Pareto improvement. Thus, under both rules the party without the option may end up, ex-post, worse off. However, unlike rule 2, rule 6 violates the ex-post individual rationality constraint as the polluter is forced to pay damages even if he prefers to shut down his plant. Lastly, both rules accommodate only one party's potential risk-aversion.

4. Modular liability rules

4.1. Introduction

All the previous assumptions are maintained. As an example of MLRs, consider a case where the resident gets a put option to force a transfer of the entitlement, as under rule 6, but the polluter has a consecutive put option to sell the entitlement back (as under rule 5). As the MLR is a compound of rules 5 and 6, it is called rule 6 + 5.

The motivation for rule 6 + 5 is that in many real-life settings the polluter cannot be forced to operate his plant, even if the resident exercises her put option. The polluter effectively

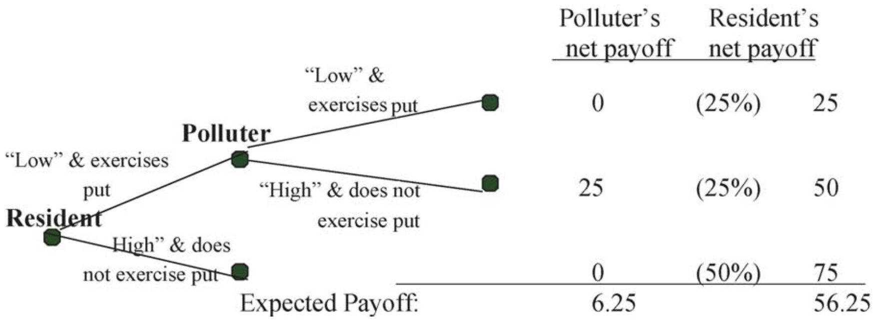

Fig. 3. Modular liability rule 6 + 5.

has the choice to close down his plant, which in our model is equivalent to exercising back his consecutive put option.[19] The polluter will acquiesce to the resident's put option if he is "high." Otherwise, he will shut down his plant.[20] Effectively, a "high" polluter will hold the entitlement only if the resident agrees.

4.2. Allocation in the shadow of rule 6 + 5

When applying MLR 6 + 5 courts should first set the exercise price. The analysis below is limited to a case where the exercise price set by the court is identical for both options.[21] In the appendix it is proved that the optimal exercise price, under the present assumptions, equals the parties' *expected* valuation. An important assumption that drives this result is that the parties' distributions are identical. Fig. 3 shows the parties' possible actions under this MLR when the exercise price, for both options, is 50:

Fig. 3 reveals that under rule 6 + 5 the polluter's expected payoff is 6.25[22] and the resident's expected payoff is 56.25.[23] Chart 4 presents this graphically.

[19] *See* Epstein at page 843, recognizing that there is no "duty to pollute." As already mentioned *supra* note 11 Krier and Schwab were first to suggest and immediately to reject rule 6 as feasible for exactly the same reasons. *See Another Light supra* note 8 at note 92 (arguing that the polluter under rule 6 holds a trump: it "could simply avoid rather than pay [its] costs of avoidance"). In this sense, rule 6 violates the third constraint, described above, for a "realistic" rule: the polluter's ex-post individual rationality. Rule 6 + 5, in contrast, does not violate it.

[20] Krier and Schwab's rule 5 suffers from a different disadvantage. As Levmore pointed out, rule 5 may penalize the victim. Levmore *supra* note 8 at 2166–67 ("if a judge follows Krier and Schwab and allows the [polluter] to continue on as before or instead choose to stop but then collect [the resident's] (projected but not yet suffered) damages [the resident] may well be sorry that she brought her complaint."). Indeed Krier and Schwab themselves were aware of this criticism, *See Another Light supra* note 3, at 474–75.

[21] For an analysis of a regime in which the exercise price is different for the parties see Ayres and Balkin, *Legal entitlements as Auctions, supra* note 8. Ayres and Balkin's regime may be more efficient than the regime proposed in this paper. However, it has a serious disadvantage (as Ayres and Balkin admit) in that it is not "Realistic" in the way defined above. Rather, it provides the parties with incentives to behave strategically. *See infra* note 50. For further criticism of Ayres and Balkin's regime, see Epstein *supra* note 7 at 2108–2111.

[22] Half of the time (when the resident does not exercise her put option) he gets 0 and half of the time (when the resident does exercise her put option) the polluter either gets 0 (if he is "low" and exercises his consequent put option) or gets 25 (if he is "high").

[23] Half of the time (when she is "high") she does not exercise her put option, in which case she has on average 75, and half of the time (when she is "low") she does exercise her put option and gets 50 unless the polluter

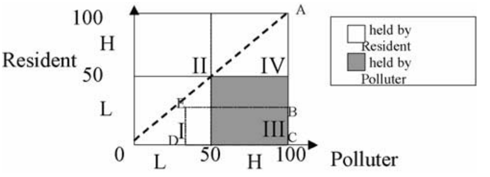

Chart 4. “Modular” liability rule 6 + 5.

As Chart 4 shows, the polluter will hold the entitlement if, and only if, he is high and the resident is low. The following lemma can be stated:

Lemma 4. *In the symmetric world, allocation in the shadow of liability rule* 6 + 5 *yields the following results*: (a) *the optimal exercise price* (D^*) *equals the expected valuation of either party*, (b) *when setting the damages to equal* D^*, *the joint ex-ante payoff*, J^*, *reaches the second-best solution*, *and* (c) *each party extracts half of the surplus generated by this rule*.

Remarks. (a) Proofs of all parts are located in the appendix.
(b) Under our assumptions $D^* = 50$, $E[V_r] = 56.25$, $E[V_p] = 6.25$, and $J^* = 62.5$.
(c) Intuition. For part (a): The social planner needs to set D^* to maximize the number of transfers from a low resident to a high polluter; all of which are efficient transfers. Geometrically, observe in Chart 4 that every exercise price, D, the social planner can set is represented in a rectangle blocked within triangle OAC.[24] For part (b): rule 6 + 5 does not reach the first-best result despite that all of the transfers made in its shadow are efficient.[25]

exercises his consequent put option, in which case the resident is stranded with the entitlement that is worth to her, on average, only 25. To see how setting two different exercise prices leads to strategic behavior, consider a case in which the exercise price for the resident’s put option is 50 whereas the exercise price for the polluter’s exercise price is 25. In this setting, a resident whose private valuation is above 50 and who otherwise will not exercise her put option may well decide to exercise her put option hoping that the polluter’s private valuation is lower than 25, in which case he will exercise back his put option, so the resident will end up with entitlement to the air but also with a net welfare of 25. Within the ex-post perspective this looks like a distributional issue not an issue of efficiency, because the entitlement ends up in the hands of the highest valuer, in this case the resident. Not so. Even when ignoring the incentives that the polluter will have ex-ante knowing this will be the result and the transaction costs associated with strategic behavior, there is an efficiency loss that is generated in those cases in which the polluter’s valuation is higher than 25 but lower than 50. In these cases he will not exercise back his put option, and the entitlement will end up in the hands of the lower valuer. Mathematically, as Ayres & Balkin showed, it is possible to find an exercise price that will yield a joint ex-ante payoff that is higher than the 62.5 achieved in rule 6 + 5. However, this strategic behavior “presents serious problems in practice.” *See supra* note 45.

[24] For example, if the social planner had set $D = 30$, cell III would have been rectangle BCDE. The task of the social planner, thus, is to maximize the area of the rectangle blocked within triangle OAB. Indeed the maximum rectangle area is the square achieved when $D^* = 50$.

[25] Every time we observe a transfer (*See* cell III) we know (in contrast to rule 2 and rule 6) that it is from a low resident to a high polluter, and thus efficient.

This is because, as Chart 4 shows, there are inefficient non-transfers.[26] For part (c): In a symmetric (and uniform) world, as both parties have the same distributions, the resident will be low and sell 50% of the time in which case she either gets damages equal to her expected valuation or keeps an entitlement equal to the average of her expected valuation and her minimum endpoint. The other 50% of the time, when she is high, her expected valuation equals the average between her expected valuation and her maximum endpoint. Thus, in total, her payoff above her mean is equal to one-eighth of r, which is half of the surplus. The polluter is guaranteed of a positive payoff because he is guaranteed the average of the difference between his maximum estimated valuation and his expected valuation, which is also equal to one-eighth of r.

(d) Distribution perspective. Polluter: His expected payoff increases from 0 to 6.25 since, under rule 6 + 5, he receives an option he did not have under rules 1 or 6. Resident: Under rule 6 + 5 she receives an option not present in rule 1 and her expected payoff increases from 50 to 56.25, representing the value of the option to force a transfer whenever she is low. Her expected payoff did not increase by 12.5 as was the case in rule 6 because the option is subject to a forced buy back from a low polluter.
(e) Ex-ante welfare analysis. Interestingly, harnessing both parties' private information did *not* raise the total payoff beyond the second-best solution of 62.5, which was achieved under both rule 2 and rule 6. This is because we assume that both parties have identical distributions and private information, thus, harnessing the polluter's private information means effectively that we allow him to dictate a transfer from one low party to the other. From an ex-ante perspective this transfer creates no value.[27] This point is shown by comparing Chart 4 with Chart 3 to observe that the only difference between rule 6 + 5 and rule 6 is in cell I, which has, within the ex-ante perspective, only distributional importance.[28]
(f) Efficiency criteria. Rule 6 + 5 Pareto dominates rule 1 within the ex-ante and ex-post perspective. The rule exploits the ex-post gain from the otherwise impossible trade between a "low" resident and a "high" polluter. In contrast with rules 2 and 6, switching from rule 1 to rule 6 + 5 does not have the guaranteed adverse effect of one of the parties being better off at the expense of the other. A low polluter may not be even ex-post worse off.
(g) Division of the surplus. Unlike rules 2 and 6, which allocate the entire surplus to the option holder party, rule 6 + 5 divides the surplus equally between them.

[26] No-transfer can happen for two reasons. First, because the resident, whose valuation is higher than 50, did not exercise her put option. In that case, it may well be that the polluter's valuation is even higher, and thus inefficient (*See* the southeast triangle in cell IV). Second, the resident did exercise her put option but the polluter, whose valuation is also lower than 50, exercises back his. In that case it may well be that the valuation of the low polluter is even higher than that of the low resident (*See* the southeast triangle in cell I).

[27] Thus, the surplus of 12.5 that was generated under rule 6 + 5 is an artifact of the assumption that both parties' distributions are *identical*. Later on, however, this symmetry assumption is relaxed and it is shown that, holding the means of the parties' distributions constant, rule 6 + 5 yields a *lower* joint payoff than rule 6 and rule 2. *See infra* Lemma 4b.

[28] Unless this cell could be split along the diagonal. The same analysis applies to a comparison of rule 6 + 5 with rule 2 with respect to cell IV.

(h) Risk aversion. Unlike rule 2 and rule 6, which created difficulties to risk-averse residents and polluters, rule 6 + 5 guarantees both parties a minimum expected payoff that is higher than the payoff they may get under rule 1, without the risk of getting less ex-post payoff. Consequently, rule 6 + 5 is preferred to rule 1, even when *both* parties are risk averse.

(i) Comparison with regular liability rules. Rule 6 + 5 is the only rule that does not violate any of the parties' ex-post individual rationality constraints described above, which is also an ex-post Pareto improvement over property rule 1 and, lastly, which is preferred to rule 1 even when *both* parties are risk averse.

    Each of the three rules considered here yield the second-best ex-ante solution of 62.5 by harnessing the option-holders' private information. Indeed, all three rules allocate surplus to the option holder. Rule 6 + 5, however, by granting options to both sides, harnesses both sides' private information and thus divides the surplus between them. This gives the social planner more flexibility in achieving distributional goals without harming the allocative efficiency goal. As the next section shows, with other MLRs, there is even more flexibility than that.

(j) Family of rules. Rule 6 + 5 is not the only rule that within the ex-ante perspective could be described by Chart 4 above. Rule 2 + 4 (in which the resident holds the entitlement and both parties have a call option) yields the same joint payoff and has the same distributional consequences.[29] In a similar manner, a whole series of equivalent rules exist for cases where it is the polluter who holds the entitlement. For example, rule 5 + 6 (which is similar to rule 6 + 5 but for the fact that it is the polluter this time who get to hold the entitlement) also yields a joint ex-ante payoff of 62.5, but this time the polluter receives 56.25 and the resident 6.25. A more complete analysis of this matter could be found elsewhere.[30]

Proposition 1. *When parties have symmetric distributions of valuations*:

(a) All liability rules and modular liability rules can attain, with the appropriate exercise price, the second-best result.

(b) The rules may vary with respect to their distributional consequences. Specifically, regular liability rules grant all the surplus to one party, but modular liability rules equally divide it between them.

(c) For both liability rules and modular liability rules, it is possible to find rules that do not violate the parties' ex-post individual rationality constraint.

5. Extensions to the basic model

This section continues to compare property rules, liability rules and MLRs, with rule 6 + 5 serving as the paradigmatic MLR. Here, we will relax the symmetric ignorance assumption in three ways. First, we will assume the parties' distributions have the same means but

[29] Ellickson observed this rule many years ago. Ellickson, *Alternative to Zoning: Covenants, Nusiance Rules, and Fines as Land Use Controls*, 40 U. Chi. L. Rev. 681, 738 (1973).

[30] *See* Section 4.3 in the SSRN version of this paper.

different variances (a mean-preserving world). In this world, MLRs achieve a lower joint payoff than the best "regular" liability rules. Second, we will assume that the parties' distributions do not have the same means but do have the same variances (a variance-preserving world). In this world, MLRs may achieve a higher joint payoff than the regular liability rules. Lastly, we will explore an unconstrained world.

5.1. A mean-preserving world—a framework for comparing the rules

All the previous assumptions are maintained, except it is assumed here that the parties' distributions are not identical so that $p < r$. Specifically, it is assumed that the resident's private valuation is uniformly distributed between 0 and 100, but that the polluter has *less* private information so his valuation is uniformly distributed in the interval $[50 - p, 50 + p]$, for $0 < p < 50$.[31] The next sections compare the allocative efficiency of the various rules.

5.2. First-best allocation

As before, the first-best solution is achieved when the entitlement is allocated to the highest valuer. Chart 5 presents this graphically.

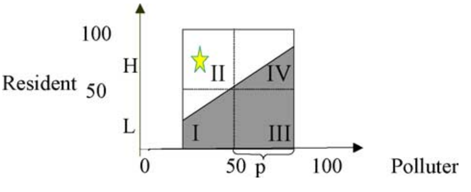

Chart 5. First-best possible allocations of entitlement in the mean preserving world.

Notice that the polluter's distribution is symmetrically narrower than the resident's. Under the line is the area where the polluter should receive the entitlement as the highest valuer. Conversely, the area above the line is where the resident should receive the entitlement. The appendix shows that here the first-best yields a joint payoff of $J_{\text{fb}} = 62.5 + p^2/600$.[32]

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[31] A motivation for the polluter's lower-variance assumption is that often it is easier for the court to estimate the polluter's valuation than to estimate the resident's because the polluter may be an experienced commercial firm with business plans and financial reports that provide the court with quantifiable information about the polluter's valuation. The resident, in contrast, may be an individual whose valuation is deeply subjective and highly private.

[32] Observe that the first-best joint payoff reaches its maximum of 66.66 when $p = 50$. This is the case of symmetric distributions that we studied in the previous section. In contrast it reaches its minimum of 62.5 when $p = 0$, which is the case of one-sided incomplete information. In that case, as we will see below, liability rules *can* reach the first-best result. Thus, the only case where the maximum attainable joint payoff by a liability rule coincides with the first-best solution is in a world of one-sided incomplete information. The fact that the first-best solution increases with '*p*' seems to be a surprising result. Yet, in a two-sided world, where the polluter's distribution is, say, between 40 and 60, two opposite effects happen. First, there is an ex-ante welfare *increase* due to the increase in the top range of the polluter's distribution (the part of his distribution between 50 and 60). The second effect is that there is a welfare *decrease* due to the bottom range of the polluter's distribution (the part between 40 and 50). Interestingly, the gain from the top range always exceeds the loss from the bottom range. To see it, assume

5.3. Allocation in the shadow of property rule 1

As before, under property rule 1 the resident is expected to retain the entitlement, which is worth on average, 50. The polluter never pollutes and has a welfare of 0. Thus, ex-ante, the joint welfare equals the resident's expected valuation. Lemma 1 applies to this mean-preserving world, too.

5.4. Allocation in the shadow of liability rule 6

The parties' possible "game tree" is exactly as was shown in Fig. 1. In addition, Lemma 3 above applies in full, and where the joint payoff reaches the second-best, the resident's payoff is equal to the joint payoff and the polluter's payoff is zero. Chart 6 shows that whenever the resident is "low," she forces a transfer.

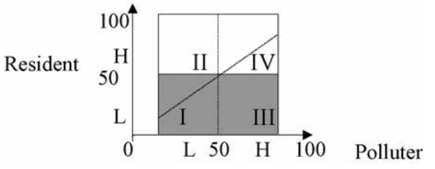

Chart 6. Liability rule 6 in a mean preserving world.

Chart 6 shows that whenever the resident is "low", she forces a transfer. Observe that rule 6 yields the second-best solution. The intuition for it is that, as Fig. 2 above shows, the polluter is not part of the "game" under rule 6, so changing the assumptions about his distribution does not change the result achieved.

for the moment that the polluter's valuation is always 50 so that he has no private information at all. In this world of one-sided incomplete information, the court should give the entitlement to the resident if her private valuation is above 50, and to the polluter if below. Once we assume a two-sided world, so that the polluter's distribution becomes symmetrically wider, say between 40 to 60, two opposite effects happen. First, there is an ex-ante welfare *increase* due to the increase in the top range of the polluter's distribution (the part of his distribution between 50 and 60). For example, assuming that the resident's valuation is 40, unlike in the one-sided incomplete information case, where the highest valuer was the polluter with a valuation of 50, in the two-sided case, ex-ante, there is a 50% chance that the polluter's valuation will be even higher, between 50 and 60. From the ex-ante perspective it means a 50% chance for a welfare increase of 5. The second effect is that there is a welfare *decrease* due to the bottom range of the polluter's distribution (the part of the distribution between 40 and 50). Specifically there is a 50% chance (ex-ante) that the polluter's valuation will be lower than 50, between 40 to 50. From the ex-ante perspective it means a 50% chance for a welfare decrease of 5. As this example demonstrates, one might expect that these two effects would cancel each other out, so ex-ante, there will be no welfare benefit. This intuition is wrong. To see it, assume now that the resident's valuation is 48. Again, the top range of the polluter's distribution generates a 50% chance for a welfare increase of 5. However, the bottom range of the distribution does not generate a symmetric 50% chance of welfare decrease of 5. Rather, if the polluter's valuation is lower than 48, the court will allocate the entitlement to the resident instead; the loss from the polluter's bottom range is capped by the resident's valuation. Formally put, whereas the ex-ante welfare gain is $p/2$, the welfare loss is min[$p/2.50 - \mathrm{Vr}$], and the latter expression is always less than the former. (It should be noted that an equivalent analysis applies when we assume that the resident's valuation is above 50.) This example helps us to understand why the more dispersed the polluter's valuation is, the higher the ex-ante payoff. The reason is that the gain from the top range always exceeds the loss from the bottom range.

5.5. Allocation in the shadow of liability rule 2

The parties' possible "game tree" is exactly as was shown in Fig. 1. However, here the joint welfare is, within the ex-ante perspective, not equal to the second-best solution of 62.5. Chart 7 represents the ex-ante perspective of rule 2.

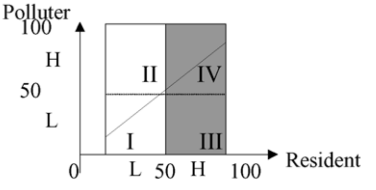

Chart 7. Liability rule 2 in a mean preserving world.

Chart 7 demonstrates that whenever the polluter is "high" (cells III and IV) he holds the entitlement. The following lemma can be made:

Lemma 2*. *In the optimal mean-preserving asymmetric world, allocation in the shadow of liability rule 2 yields the following results*: (a) *the optimal exercise price* (D^*) *equal the resident's* (*and the polluter's*) *expected value* $D_2^* = \mu$; (b) *when setting the damages to equal* D_2^*, *the joint ex-ante payoff is* $J_2^* = \mu + p/4$, *which does not reach the second-best solution*; (c) *the resident's expected payoff equals her expected valuation*: $V_{r_2} = \mu$; (d) *the polluter's expected payoff equals one quarter of p*: $E[V_{p_2}] = p/4$.

Remarks. (a) Proofs of all parts are given in the appendix. (b) Note that Lemma 2* is a generalization of Lemma 2 to a mean-preserving world; however, as '*p*' no longer equals 50, rule 2 does not yield the second-best solution. The intuition is that as rule 2 harnesses only the polluter's private valuation, and as the polluter has narrower distribution, there is less gained from harnessing his private information than from harnessing the resident's, as rule 6 does.

5.6. Allocation in the shadow of liability rule 6 + 5

Under MLR 6 + 5 the parties' possible actions are exactly as was shown in Fig. 3. Interestingly, the optimal exercise price D_{65}^* does not equal either of the parties' expected valuations as was the case above. Fig. 4 depicts the optimal exercise price, D_{65}^*, as a function of the polluter's private information.

As Fig. 4 shows, to the extent that $p < 16.67$ (which means that the polluter's distribution is narrower than [33.33, 66.67]) the optimal exercise price D_{65}^* is equal to the lower endpoint $50 - p$. Otherwise, when the polluter's distribution is wider, the optimal exercise price equals half of the upper endpoint.[33] Lemma 4*a states this formally.

[33] It can readily be seen that when $p = 50$ (which reflects the world of *symmetric* incomplete information), the optimal exercise price is indeed 50. In a similar manner, if $p = 0$, it essentially means that the polluter has no private information at all (the world of one-sided incomplete information) in which rule 6 + 5 collapses to rule 6 and indeed the optimal exercise price is again 50.

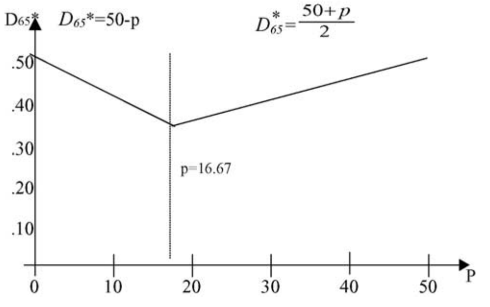

Fig. 4. Optimal exercise price as a function of the polluter distribution in a mean preserving world.

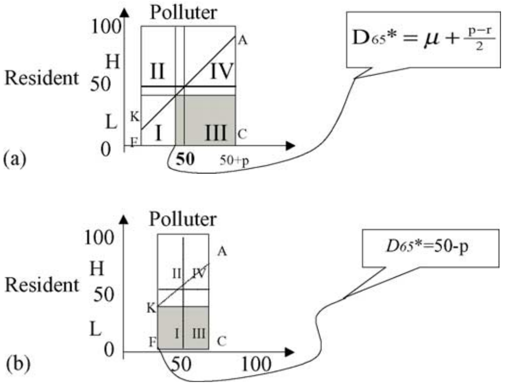

Chart 8. (a) Liability rule 6 + 5 for a wide polluter's distribution in a mean preserving world. (b) Liability rule 6 + 5 for a narrow polluter's distribution in a mean preserving world.

Lemma 4*a. *In the mean-preserving world, allocation in the shadow of liability rule* 6 + 5 *requires the optimal exercise price* (D^*) *to be equal to*:[34]

$$D^*_{65} = \begin{cases} \mu + \frac{p-r}{2} & \text{if} \quad \frac{r}{3} < p < r \\ \mu_\mathrm{p} - p & \text{otherwise} \end{cases}$$

Remarks. (a) As under our current assumptions $r = 50$, for the regular case, the optimal damages equal half of the polluter's maximum estimated valuation. (b) Intuition. *For first formula* (the 'regular case'): In the symmetric world, D^* was set to maximize the number of efficient transfers.

As Chart 8a shows, the division of the cells under rule 6 + 5 is no longer symmetrical. The maximum blocked rectangle is a square, whose sides equal half of the sides of triangle OAC, and the triangle sides equal $(50 + p)$, $D^*_{65} = 25 + p/2$.

[34] More generally, D^* equals the average between the minimum estimated valuation of the resident and the maximum estimated valuation of the polluter.

Observe that as p increases, D^* increases. Indeed, when $p = 50$ we are back in the symmetrical world where $D^*_{65} = 50$. An intuition for the lower D^* when $p < 50$ would be as follows. As we saw, the optimal exercise price for regular liability rules, say rule 6, was equal to the mean of the passive party, the polluter, resulting in the best price, given that there is one round of transfers. In MLRs, as both parties signal their valuations, the social planner has to construct one exercise price that is relevant to two consecutive rounds. Accordingly, the relevant range to partition would be the range between the resident's lower endpoint and the polluter's upper endpoint.

At some point, as p decreases D^* may fall outside of the polluter's *lower* endpoint, in which case the "otherwise" formula applies. Consequently, in the mean preserving world, the first formula applies only when the parties' distributions are not more than three times wider than each other.

$$D^*_{65} = \tfrac{1}{2}[(\mu\mathrm{p} + p) + (\mu\mathrm{r} - r)], \quad \forall r < 3p \text{ and } p < 3r$$

For the second *formula* (the 'otherwise case'): Observe that this is a corner solution of the first part. Specifically, if the polluter's distribution is relatively narrow, say between 40 and 60, the optimal exercise price D^* should have been 30. However, this would have been lower than the polluter's lower endpoint. Graphically, as Chart 8b below shows, the maximum rectangle blocked in the trapezoid ACFK is the rectangle compound from cells I and III.

The intuition is that when D^*_{65} = any number below the polluter's lower endpoint, the polluter never exercises back his consecutive put option because his private valuation is always higher. Increasing D^*_{65} from 30 to 40 does not result in the polluter exercising his option, but it increases the total welfare as it allows the court to screen the residents whose private valuation was between 30 and 40. As for the joint payoff, the following lemma can be stated:

Lemma 4*b. *In a mean-preserving world, allocation in the shadow of liability rule* 6 + 5 *yields the following results*: (a) *when setting the damages to equal* D^*_{65}, *the joint ex-ante payoff,* J^*, *does not reach the second-best solution,* (b) *if* $r/3 < p < r$, *each party extracts half of the surplus generated by this rule*.

Remarks. (a) Proofs of all parts are given in the appendix. (b) Intuition. Part (a): Although rule 6 + 5 harnesses *both* parties' private information to the process of allocating the entitlement between them, it did *not* yield the second-best solution which was achieved by rule 6. The reason is that the court knows that the polluter's private valuation is distributed in a narrower range then the resident's. Thus, there is less gained from harnessing the polluter's private information than from harnessing the resident's. Because more is gained from harnessing the resident's private information, rule 6 + 5 yields worse results than rule 6. Part (b): If there are no corner solutions, rule 6 + 5 equally divides the surplus between the parties. If there are corner solutions, because D^*_{65} is shifted inwards, the surplus is not evenly split. (c) Comparison of the rules. Efficiency: under our assumptions and from an allocative efficiency perspective, rule 6 dominates rule 6 + 5, which, in turn, dominates rule 2. Fig. 5 summarizes:

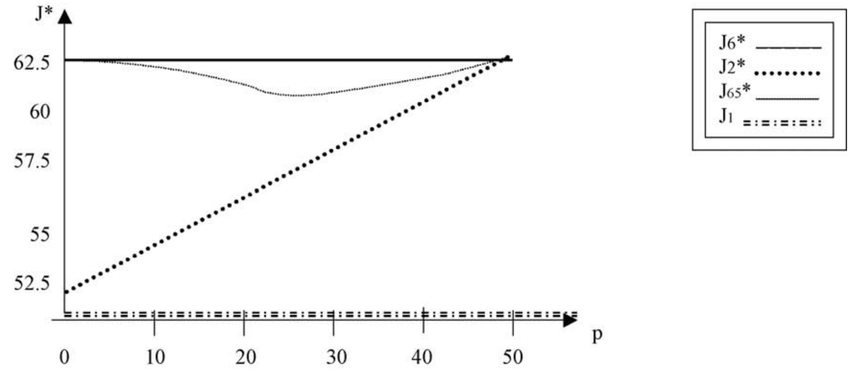

Fig. 5. Comparison of the joint payoff as a function of the polluter's distribution, for the different rules.

(d) Comparison of the rules. Distribution. Resident: She prefers rule 6 where she gets a constant ex-ante payoff of 62.5. Next, she prefers rule 6 + 5, which provides her with payoffs between 62.5 and approximately 55, depending on the distribution of the polluter. Lastly, under rules 2 and 1, she receives a constant expected payoff of 50.

Polluter: One may plausibly conjecture that the polluter will always prefer rule 2 because he dictates whether he gets the whole surplus, as opposed to rule 6 + 5 where he gets, most of the time, just half of the surplus. What this overlooks is that rule 6 + 5 usually generates more surplus than rule 2. Thus, for some values of p, (approximately when $0 < p < 21.5$), the polluter is better off having a fraction of a bigger surplus than the total of a smaller one. In any case the polluter always prefers either rule 2 or rule 6 + 5 to rule 6 or 1, in which he has an ex-ante welfare of only 0.

(e) Family of rules. As in the symmetric world, the social planner has a choice of rules which achieve the same joint payoff, yet with a different distribution.[35]

The following proposition can be made:

Proposition 2. *In the mean-preserving world a welfare-maximizing social planner should apply the rules in the following way*: (a) *If both parties have the same uniform distribution, then all liability rules* (*including MLRs*) *yield the same joint payoff*, (b) *if the polluter has more private information, then rules* 2 *and* 5 *maximize the joint ex-ante payoff* (*regular liability rules dominate*), (c) *if the resident has more private information*, *rules* 6 *and* 4 *maximize the joint ex-ante payoff* (*again*, *regular liability rules dominate*), (d) *in the absence of lump-sum transfer payments, MLRs can achieve a greater variety of distributional goals with less decrease in the joint payoff than the non-maximizing liability rules, and* (e) *to that*

[35] Thus, rule 6 is not the only joint payoff maximizing rule. Rule 4, too, achieves the same joint payoff but with a different distribution. The common characteristic of these rules is that they all harness only the resident's private information. Then come MLRs (rules 6 + 2, 6 + 5, 5 + 4, 5 + 6) which harness *both* parties private information. For the same reasons explained in comment (b) above, they all yield a lower joint payoff than the first group. Next are the rules which harness only the polluter's private information (rules 2 and 5), which yield even a lower joint payoff. Lastly are presented the property rules, which yield the lowest joint payoff. Table 1 which compares the rules is on file with the author.

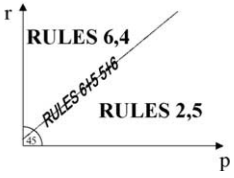

Fig. 6. Efficient rules in the mean-preserving world.

end, the social planner should set the exercise price equal to the average of the relevant parties' maximum and minimum endpoints, unless the parties' distributions are three times larger than each other in which case the exercise price should be set to equal to the relevant endpoint. Fig. 6 *summarizes*.

6. A variance-preserving world—a framework for comparing the rules

All the previous assumptions are maintained, except that now it is assumed that the parties' distributions have a different mean yet the same variance. Specifically, it is assumed that the resident's private valuation is uniformly distributed in the interval $[\mu_r - \pi, \mu_r + \pi]$ and that the polluter's private valuation is uniformly distributed in the interval $[\mu_p - \pi, \mu_p + \pi]$.

6.1. First-best allocation

As before, the first-best allocation is achieved when the entitlement is allocated to the highest valuer. Chart 9 represents a case where the resident's distribution is along the interval [25,100] and the polluter's valuation is distributed along the interval [50,125]. As before, the diagonal (parts of it are now imaginary) represents an indifference locus that divides the square into two possible allocations.

6.2. Allocation in the shadow of (regular) liability rules

We saw that in the mean-preserving world, the choice between paradigmatic rule 2 and 6 depends on the amount of private information each party has. In the variance-preserving

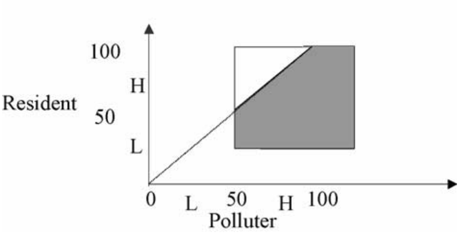

Chart 9. First-best possible allocations of entitlement in a variance preserving world.

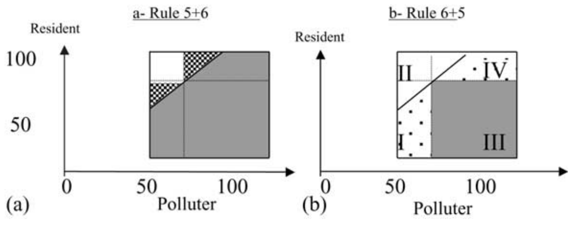

Chart 10. Rule 5 + 6 vs. rule 6 + 5 in a variance preserving world.

world, as both parties have the same amount of private information, both rules yield the same joint payoff. The next lemma can be made:

Lemma 5. *For regular liability rules in the variance-preserving world*: (a) *the optimal damages equal the passive party's expected valuation* $D_6^* = \mu_{\mathrm{p}}$, $D_2^* = \mu_{\mathrm{r}}$, *and* (b) *all regular rules yield the same joint payoff,* J^**, but with a different distribution of payoffs.*

Remarks. (a) Interestingly, when the difference between the parties' means is greater than π, $J'' = \max[\mu_{\mathrm{p}}, \mu_{\mathrm{r}}]$.[36]

6.3. Allocation in the shadow of modular liability rules

Under this world, modular rule 6 + 5 and 5 + 6 do not yield the same joint payoff anymore. Recall that rule 6 + 5 facilitates a transfer to the polluter whenever the polluter is high and the resident is low. Rule 5 + 6, in contrast, facilitates a transfer from the polluter (who is allocated with the entitlement) to the resident, whenever the polluter is low and the resident is high. Chart 10 presents these rules graphically:

Chart 10b represents rule 6 + 5. The dotted section indicates the areas of inefficiency. There, the polluter should have been the holder, but it is the resident who holds the entitlement. Chart 10a represents rule 5 + 6. Here, by construction $D^* = 75$.[37] Thus, the polluter will exercise his put option whenever he is low. Next, the resident has a consecutive put option, which she will exercise when she, too, is low. As a result, the resident will hold the entitlement if she is high and the polluter is low, which is represented by the white square. Here, the inefficiency areas are the gray-dotted areas where the resident should have been the holder. As the inefficiency areas in Chart 10a are smaller than the ones in Chart 10b, rule 5 + 6 is, in this case, yields higher joint payoff than rule 6 + 5. (Recall, that in the

[36] To see it, consider, for example, the case where when the polluter's distribution is between 40 and 80 and the resident's between 70 and 110, the resident's mean of 90 is larger than the polluter's upper endpoint of 80, and thus, on average, higher than any of the polluter's valuations, even than his highest one. In that case the court should allocate the entitlement to the party with the highest mean, the resident in this case, which, in her turn, will never exercise her put option (for paradigmatic rule 6). Observe that regular liability rules are not able to make nuanced allocations in the intersection between the resident's lower endpoint of 70 and the polluter's upper endpoint of 80.

[37] In the appendix it is proved that D^* for both rule 5 + 6 and rule 6 + 5 in the variance-preserving world is the average of the parties' means. For the above distributions $D^* = 1/2 \times (62.5 + 87.5) = 75$.

mean-preserving world, these two areas were always identical.) The following lemma can be stated:

Lemma 6. *For modular liability rules in a variance-preserving world*: (a) *the optimal exercise price* (D^*) *is equal to the average of the parties' means,*

$$D^*_{56} = D^*_{65} = \tfrac{1}{2}(\mu_\mathrm{r} + \mu_\mathrm{p}), \quad \forall|\mu_\mathrm{p} - \mu_\mathrm{r}| < 2\pi$$

and (b) *all modular rules do not yield the same joint payoff, unless the difference between the parties' means is greater than* 2π, *in which case they do.*

Remarks. Intuition. For part (a): the optimal damages equal the average of the parties' means. This is a corollary from the general principle that the optimal damages, for rule 5 + 6, equal the average of the polluter's lower endpoint and the resident's upper endpoint. For part (b): when the difference between the means is larger than the full dispersion (so parties' distributions do not intersect), as it is when the polluter's distribution is between 40 and 80 and the resident's is between 90 and 130, the court should, and easily could, allocate the entitlement to the party with the highest mean (which will never exercise its option), because this party is *always* the highest valuer.[38]

6.3.1. Choosing among the rules

Two questions naturally arise. First, when does rule 5 + 6 yield a higher joint payoff than rule 6 + 5. Second, when does the superior modular liability rule yield a higher joint payoff than the regular liability rules.[39] As for the first question the following proposition is proved in the appendix:

Proposition 3. Under the variance-preserving world, rule 6 + 5 yields a higher joint payoff than rule 5 + 6 if and only if the resident's mean is higher than the polluter's.

Remark. (a) Intuition. Observe that rule 6 + 5 over-allocates the entitlement to the resident, whereas rule 5 + 6 over-allocates the entitlement to the polluter. This could be easily seen by observing the inefficiency areas in Chart 10a and b. Thus, as the polluter's mean increases it is better, on average, to over-allocate to him (as rule 5 + 6 does) than to over-allocate to the resident (as rule 6 + 5 does).[40]

As for the second question, Proposition 4 summarizes: in the variance-preserving world, the following holds: (a) if there is some intersection between the parties' distributions, the optimal modular liability rules are superior to (regular) liability rules; and (b) if there is

[38] Observe that whenever there is intersection between the parties' distributions, unlike regular liability rules, modular liability rules *are* able to make some nuanced allocations in the intersection between the resident's lower endpoint and the polluter's upper endpoint.

[39] We saw above that in the variance-preserving world, all (regular) liability rules yield the same joint payoffs, so the choice of which regular liability rule is superior is avoided.

[40] It is better to allocate the entitlement to the party with the higher mean because this party will have, on average, higher welfare. Accordingly, rule 5 + 6 which allocates cells I and IV to the polluter will be more efficient when the polluter's mean is higher, as drawn in Chart 10.

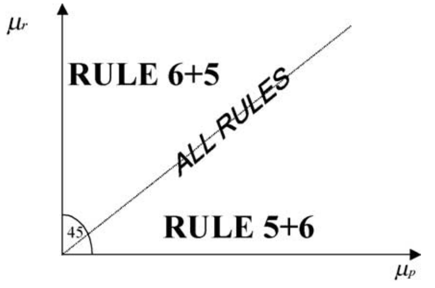

Fig. 7. Efficient rules in the variance-preserving world.

no intersection between the parties' distribution, the optimal modular liability rules are equivalent to the (regular) liability rules; all yield a joint payoff that is equal to the higher valuer's mean.

Remarks. Intuition. The fact that in the variance-preserving world modular liability rules are superior is a corollary of Proposition 5(I) (c). Thus, the reader is referred to comment (b) of that proposition. Fig. 7 summarizes.

7. The unconstrained-world

We have identified two rules-of-thumb: in a mean-preserving world regular liability rules dominate and, among these rules, the ones that harness the party with more private information are superior. In a variance-preserving world, MLRs dominate and the ones that over-allocate to the party with the higher mean, are superior.[41]

In real life situations, however, the social planner may well encounter parties' whose valuations differ both in their means and variances. In this unconstrained world, generalized results from previous lemmas hold:

Lemma 7. *When parties' valuations differ both in their means and variances the following holds*:

(a) For regular liability rules, the optimal damages equal the passive party's expected valuation. $D_6^* = \mu_\mathrm{p}$, $D_2^* = \mu_\mathrm{r}$ (Kaplow and Shavell, 1996; Ayres, 1998).
(b) For modular liability rules:

$$D_{65}^* = \begin{cases} \frac{(\mu_\mathrm{r}-r)+(\mu_\mathrm{p}+p)}{2} & \forall \mu_\mathrm{r}-\mu_\mathrm{p} > p-3r, \quad \mu_\mathrm{r}-\mu_\mathrm{p} > r-3p, \quad \mu_\mathrm{r}-\mu_\mathrm{p} < r+p \\ \mu_\mathrm{r}+r & \forall \mu_\mathrm{r}-\mu_\mathrm{p} \le p-3r \\ \mu_\mathrm{p}-p & \forall \mu_\mathrm{r}-\mu_\mathrm{p} \le r-3p \end{cases}$$

[41] Although I refer here to an "unconstrained world," the reader should be reminded that the model assumes that parties' valuations are uniformly and independently distributed.

$$D^*_{56} = \begin{cases} \frac{(\mu_r - r) + (\mu_p + p)}{2} & \forall \mu_r - \mu_p < 3r - p, \quad \mu_r - \mu_p < 3p - r, \quad \mu_p - \mu_r < r + p \\ \mu_r + r & \forall \mu_r - \mu_p \geq 3p - r \\ \mu_p - p & \forall \mu_r - \mu_p \geq 3r - p \end{cases}$$

Remarks. (a) Proofs of all parts are given in the appendix. (g) Intuition. For part (b): as we saw in comment (b) of Lemma 4* above, the ingenuity of MLRs is that they manage to isolate the areas where parties' private information is relevant. However, sometimes the mechanism of MLRs yields a D^* that falls outside of either (or both) of the parties' endpoints (there are four such possibilities). These are the cases where the corner solutions apply.

The most important task remains—the choice among the different rules. Here, the rules-of-thumb formalized in Propositions 2 and 3 apply. Next, as before, the social planner needs to choose between the superior regular liability rule and the superior MLR. The choice now depends on the parties' differences of means, differences of variances, and the relationship between the latter two. The next proposition is proved in the appendix:

Proposition 5. When parties' valuations differ both in their means and variances the following holds:

(I) If there is some intersection between the parties' distributions then:

(a) For regular liability rules: If the difference between the parties' means is smaller than max[p,r], then rule 6 is superior to rule 2 if, and only if, the resident's private information is larger than the polluter's. Formally:

$$\begin{aligned} &J^*_6 > J^*_2 \quad \text{if} \quad r > p \quad \forall|\mu_r - \mu_p| < \max[p, r] \\ &J^*_6 = J^*_2 \qquad\qquad\qquad\quad \forall \max[p, r] \leq |\mu_r - \mu_p| < p + r \end{aligned}$$

(b) For modular liability rules: rule 6 + 5 is superior to rule 5 + 6 if and only if the resident's mean is larger than the polluter's mean. Formally:

$$\begin{aligned} &J^*_{65} > J^*_{56} \quad \text{if} \quad \mu_r > \mu_p \quad \forall|\mu_r - \mu_p| > p - 3r, 3p - r, p + r \\ &J^*_{65} = J^*_{56} \qquad\qquad\qquad\quad \text{otherwise} \end{aligned}$$

(c) The optimal modular liability rule yields a higher joint payoff than the optimal regular liability rule if and only if the difference between the parties' mean (in absolute value) is larger than difference between the parties' private information (in absolute value). Formally:

$$\max[J^*_{65}, J^*_{56}] > \max[J^*_6, J^*_2] \quad \text{if} \quad |\mu_r - \mu_p| > |r - p|$$

In these cases, the surplus generated by modular liability rules is equally divided between the parties.

(II) If there is no intersection between the parties' distribution, then all rules (including property rules) yield the same joint payoff: $J^* = \max[\mu_p, \mu_r]$

which is the first-best solution.

Remarks. (a) Intuition. For part I(a): compare Proposition 2 above. The first part restates Kaplow and Shavell's and Ayres' findings. The second part represents the case where one party's mean is larger (or smaller) than the other party's endpoint. In these cases the first party, on average, has higher (or lower) valuation than any of the second party's valuations, in which case the court should allocate the entitlement to the party with the highest mean, and neither party will exercise its option.[42]

For part I(b): compare Proposition 3. The second part represents a case where one party's distribution is totally included inside the other. In this case, the corner solutions apply for both rule 6 + 5 and rule 5 + 6, and both rules yield the same joint payoff.[43]

For part I(c): observe that in general, as the difference between the means approaches zero, which resembles the mean-preserving world, the (regular) liability rules will increasingly dominate MLRs. Conversely, as the difference between the variances approaches zero, which resembles the variance-preserving world, the MLRs will increasingly dominate. Which rule dominates depends on the relative magnitude of these two effects. The surplus generated (for a uniform distribution) is equally divided between the parties because, as is proven in the appendix, whenever modular liability rules dominate regular liability rules, there is never a corner solution.[44]

For part II: if there is no intersection between the parties' distributions then any private valuation the party with the higher mean can have is larger than any private valuation the party with the lower mean can have. Thus, the entitlement always ends up in the hands of the higher valuer, and the first-best solution is achieved.

(b) Intuition revisited. To further understand the underlying intuition behind the results in the unconstrained world consider the next series of diagrams. Without loss of generality we will assume that the resident's mean is larger than the polluter' mean and that $p > r$ (which entails that rule 6 + 5 dominates rule 5 + 6 and that rule 2 dominates rule 6, respectively). The puzzle then is why rule 6 + 5 dominates rule 2 only when the difference between the parties' means is larger than $p - r$.

Fig. 8a presents a case where the difference between the parties' means is relatively large. Indeed, although the difference is smaller than $p + r$ (so that there will be intersection between the distributions), it is larger, as can be easily seen, than max[p, r] (which under our assumptions is equal to 'p').

Recall from comment (a) to Lemma 5 and comment (b) to Lemma 6 that in this situation rule 6 + 5 has an advantage over rule 2 because rule 2 is not able to make nuanced allocations in the intersection between the resident's lower endpoint and the polluter's upper endpoint (roughly the area inside the dotted ellipse). More specifically, the whole union of the parties' distributions in Fig. 8a can be divided into three areas. In the left area, it is clear that the resident is the highest valuer. In the middle areas, it is not obvious who the highest valuer

[42] Which entails that both rules yield the same joint payoff, compare remark (a) to Lemma 5. This exception is of no major significance as in those cases modular liability rules always dominate.

[43] This exception to the general rule is of no major significance because in those cases regular liability rules will always dominate.

[44] Recall from the previous proposition that modular liability rules equally divide the surplus between the parties to the extent that there are no corner solutions.

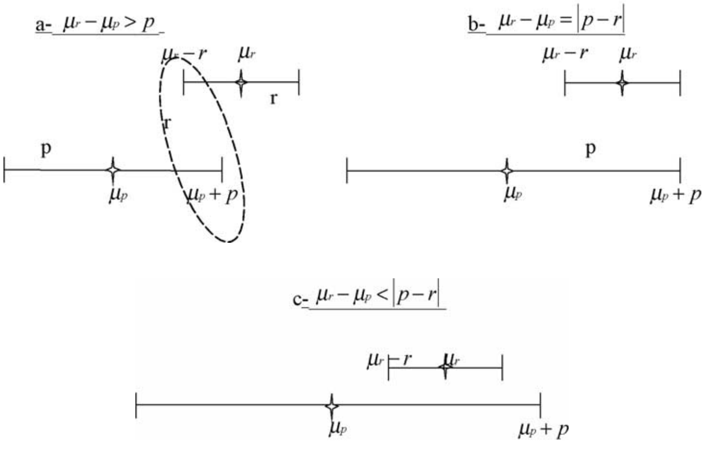

Fig. 8. (a, b) The unconstrained world, (c) the difference between the means is smaller than $p - r$.

is; this is the area inside the ellipse. Lastly, in the right area, it is clear that the resident, again, is the highest valuer.

Thus, the only region in which there is a need to make tough decisions is the middle area. Rule 6 + 5 pinpoints this area and enables efficient transfers from a low resident to a high polluter. This is done, recall, because the optimal damages for rule 6 + 5 are equal to the average of the resident's lower endpoint and the polluter's upper endpoint, which is exactly the mid-point of this middle area.

Rule 2, in contrast, is not able to make such fine tune allocations. Rather, it bluntly partitions the resident's distribution in the middle and allocates the entitlement to the polluter if his valuation is higher than the resident's mean, and otherwise allocates it to the resident. In the example depicted in Fig. 8a, the resident under rule 2 will always hold the entitlement, even in the areas inside the ellipse where, as we saw, rule 6 + 5 is able to enable efficient transfers to the polluter.

As 'p' increases (holding the difference between the parties' means (and 'r') constant) the 'middle area' that generates the rule 6 + 5 allocational advantage becomes wider and at some point is equal to the resident's distribution. Fig. 8b presents this case. Observe that now the difference between the means is exactly equal to the difference between the parties' private information ($\mu_r - \mu_p = |p - r|$). Specifically, observe that because the polluter's upper endpoint coincides with the resident's upper endpoint, both rule 6 + 5 and rule 2 partition the same area (the resident's whole distribution, in fact). Naturally, both rules have the same optimal exercise price ($D_2^* = D_{65}^*$). At this point both rules yield the same joint payoff.

As we increase 'p' even further, rule 6 + 5 is no longer superior in allocating the relevant areas for harnessing the parties' information. Fig. 8c presents this case.

Observe that as we increase 'p' we get a new area (which starts from the resident's upper endpoint and ends at the polluter's upper endpoint) where it is clear that the polluter is the highest valuer. Indeed, observe that in the situation depicted in Chart 13c, there is only one area where a tough decision should be made, this is the area that corresponds to the resident's distribution. This time it is rule 2 that is able to pinpoint the relevant area for partitioning, and it is rule 6 + 5 that misses it.

8. Summary and Implementation

In this article, I showed that harnessing both parties' private information is not always superior to harnessing just one party's private information. For uniform distributions, only when the difference between the parties' means is larger than the difference between the parties' private information, harnessing both parties' private information, via MLRs, yield higher joint payoffs than harnessing one party's private information, via regular liability rules. A further, and somewhat surprising, corollary of the analysis presented in this article was that, unlike conventional liability rules in which the optimal damages set by the court equal the aggrieved party's expected harm, in MLRs the optimal damages set by the court are, in general, equal to that mid-point of the area of intersection of the parties' distributions. MLRs pinpoint this area, where there is uncertainty about who is the highest valuer, and they enable efficient transfers from a low resident to a high polluter. Regular liability rules, in contrast, cannot make such fine tune allocations.

In order to make some practical recommendations from the above analysis, three assumptions must be maintained from the original stylized model. First, one must assume that the parties to the dispute are not able to negotiate over the court's decision. This assumption is quite realistic given that a recent study shows that parties fail in post-judgment negotiations.[45] Second, and less critical, one must assume that there is a single polluter or a single resident.[46] Third, one must assume that the courts observe parties' range of valuations, which is uniformly and independently distributed. It seems to me that few would dispute that in many real life settings parties' valuations are independent of each other. As for the assumption of uniform distributions, this is tantamount to assuming that courts are in a state of complete ignorance vis-à-vis the way parties' valuations are distributed within their range.

[45] See Farnsworth supra note 17, at 10. This is so because of the acrimony between the parties, the parties' willingness to establish reputation, the bargaining range problem, or the asymmetric information caused by the fact that each party knows its own valuation but not the other's. *See supra* notes 18 and 19. Moreover, by consciously prohibiting a post-judgment negotiation between the parties, courts can ensure that the assumption is maintained. *See* Ayres and Madison *Threatening Inefficient Performance of Injunctions and Contracts*, 148 U. of Pa. L. Rev. 45 (suggesting to eliminate plaintiffs' adverse incentives to seek an injunction in order to sell it back to the defendant and improve its payoff by, among other things, letting the defendant decide in advance whether a potential injunction against her be inalienable, in the sense that the plaintiff will not be able to sell it back to her).

[46] However, it should be noted that in many environmental disputes the residents are frequently plaintiffs in a class-action lawsuit, so there is always a rule 23 representative plaintiff who can fill the role of the resident in the model. In those cases in which it is not a rule 23 class-action lawsuit, courts can impose all sorts of preference aggregating mechanisms, such as voting, for instance. And compare Ayres *Protecting Property With Rights supra* note 3, at 822–23 ("many scholars view the numerosity problem as a trump card, suggesting that it creates an insurmountable barrier that precludes meaningful resident choice. But . . . [w]hen the residents' value of no pollution is more speculative than the factory's value of pollution and/or when residents are more risk averse, a flawed resident choice under rule 6 might be more equitable and efficient than a "perfect" polluter choice under rule 2").

I would like to suggest a practical way to implement the insights of this paper and harness parties' private information to the judicial process. Suppose that after hearing all the arguments and seeing all the evidence, the court decides that the plaintiff's valuation is uniformly distributed somewhere between $50 and $110 million, whereas the defendant's is independently and uniformly distributed between $30 and $95 million. However, the court is not certain about who the highest valuer is. The court can make the following statement:

> "After considering all the material that the parties brought to me, I decided to enable the defendant (the polluter) to continue operating his plant. However, the plaintiff (the resident) is entitled, in return, to $72.5 million dollar in compensation from the defendant for the future damages she will suffer [compensation for past damages is $X]. The plaintiff has 48 hours to announce to the court whether she agrees to accept this amount of compensation. If she does not agree, then I will order an injunction against the defendant and he will have to stop polluting. In case the plaintiff agrees to accept this sum of compensation, then the defendant has 48 hours to announce to the court whether he agrees to pay this amount of compensation. If he does not agree, then I will order an injunction against the defendant and he will have to stop polluting. If he agrees, then the defendant will pay the compensation to the plaintiff, in return for the plaintiff's consent to the continued operation of the defendant's plant."

This simple announcement achieves the same results demonstrated in the stylized model; it is conceptually the same setting that was presented in Fig. 3 above for rule 6 + 5. This simple announcement harnesses both parties' valuation to the judicial process and allows the court to make an optimal decision achieving a joint payoff of $82,920,673 rather than the $81,730,769 million it would have achieved through liability rule 2, or the $80,000,000 it would have achieved through property rule 1.

Lastly, notice that there is not necessarily a need for the court's announcement, or for the model to be sequential. Rather, the court can give 48 hours simultaneously to both parties to return and provide the court with a sealed document in which they state their decision. The presentation of a sequential model was solely for methodological reasons. This fact, of course, makes the court's announcement even simpler.

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