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PARITY, INTRANSITIVITY, AND A CONTEXT-SENSITIVE DEGREE ANALYSIS OF GRADABILITY

Yitzhak Benbaji

Larry Temkin challenged what seems to be an analytic truth about comparatives: if A is Φ -er than B and B is Φ -er than C, then, A is Φ -er than C. Ruth Chang denies a related claim: if A is Φ -er than B and C is not Φ -er than B, but is Φ to a certain degree, then A is Φ -er than C. In this paper I advance a context-sensitive semantics of gradability according to which the data uncovered by Temkin and Chang leave both statements intact.

Consider the following statement: (i) ‘For any three objects A, B, and C, if A is heavier than B and B is heavier than C, then A is heavier than C.’ This statement seems to express the analytic truth that the relation *heavier than* is transitive. Consider, secondly, a related statement, (ii) ‘For any three objects A, B, and C, if A is heavier than B, and if C is *not* heavier than B but is heavy to some degree (that is, C has some weight and is not, say, an abstract object), then A is heavier than C.’ This statement looks nearly equivalent to the former. Nevertheless, for a reason to be presented shortly, I wish to distinguish between them. I shall say that ‘heavier than’ is not only transitive, as stated in (i), but also *transitive**, as stated in (ii). Now, *all* comparatives seem, at least *prima facie*, to be both transitive and *transitive** [cf. Wheeler 1972: 320]. Hence if Φ is a graded adjective, then,

- (1a) S(emantic)-Transitivity: if A is Φ -er than B and B is Φ -er than C, then, A is Φ -er than C.
- (2a) S-Transitivity*: if A is Φ -er than B and C is not Φ -er than B, but is Φ to a certain degree, then A is Φ -er than C.

Philosophers have challenged both (1a) and (2a), either directly or indirectly. Following Stuart Rachels [1998], Larry Temkin [1996] argues against an instance of (1a) on the basis of the rationality of some intransitive preferences. He challenges the transitivity of ‘all things considered, better than’ and argues that we should not accept (1a) as a semantic, or analytic, truth. Ruth Chang [2002a, 2005] attacks (2a) by appealing to the seeming non-transitivity* of ‘more creative than’.

This paper offers a new interpretation of (1a) and (2a). On the proposed interpretation, the data uncovered by Temkin and Chang leave both S-Transitivity and S-Transitivity* intact. The paper is organized as follows: in section I, Chang's argument against S-Transitivity* and Temkin's argument against S-Transitivity are sketched. In section II, it is shown that these arguments worryingly cast doubt on the standard semantics of graded adjectives and comparatives. In section III—the main part of the paper—I shall defend (1a) and (2a) by dissolving Temkin's and Chang's puzzles.

I. Transitivity and Transitivity* Challenged

Chang plausibly argues that Mozart is neither more nor less creative than Michelangelo. More controversially, she denies that they are equally creative. Philosophers might agree. But unlike Chang, they would infer that Mozart and Michelangelo are *incomparable* with respect to artistic creativity; Mozart and Michelangelo are too remote to be compared. Others might appeal to the notion of vague or rough equality; Mozart and Michelangelo are equally creative, but the equality between them is vague or rough. Chang believes that both approaches are deeply confused. Mozart and Michelangelo are *definitely* (i.e., non-vaguely) *on a par* with respect to creativity; for her, parity is a fourth, positive, non-vague, non-derivative relation between comparable bearers of value, in addition to the relations of more than, less than, and equal to. Chang's *Small Improvement Argument* [Chang 2002a: 667 – 73] purports to establish that Mozart and Michelangelo are not equally creative. Her *Chaining Argument* [ibid.: 673 – 9] concludes that Mozart and Michelangelo are comparable with respect to creativity. She then argues that $\langle \text{Michelangelo, Mozart} \rangle$ is not a borderline case of 'more creative than'. If her arguments are sound, parity is possible; and graded adjectives that allow for parity fail to satisfy S-Transitivity* [(2a)].

The first argument is premised on the plausible assumption that Mozart is neither more nor less creative than Michelangelo. Why, then, aren't they equally creative? Imagine a possible individual, Mozart+, whose career matches Mozart's in all respects except one: Mozart+ wrote one more piece of music than Mozart wrote. Given his slightly greater productivity, Mozart+ seems to be more creative than Mozart. Is Mozart+ more creative than Michelangelo? The answer, Chang argues, must be negative; it would be extremely odd to think that Mozart would have been more creative than Michelangelo if only he had created one more piece of music. In other words, (3), Chang argues, is true:

- (3) Mozart+ is more creative than Mozart and Michelangelo is not more creative than Mozart. Even so, Mozart+ is not more creative than Michelangelo.

It follows from (3) that Mozart and Michelangelo are not equally creative; if they were, then, since Mozart+ is more creative than Mozart, Mozart+ would have been more creative than Michelangelo, too.

So much for the *Small Improvement Argument*. As far as this argument goes, Mozart and Michelangelo might be incomparable regarding creativity; the following *Chaining Argument* rules out this possibility. Imagine that there once was a very, very bad sculptor named Talentlessi. Needless to say, Talentlessi would be comparable to Mozart; after all, Mozart is very creative, whereas Talentlessi is not creative at all. Now, arguably, if A is comparable to B in some respect and A+ differs from A only slightly in this respect, then A+ also is comparable to B. According to this plausible assumption, if Talentlessi+ is only slightly more creative than Talentlessi, he, too, is comparable both to Michelangelo and to Mozart with respect to creativity. Now, there is a chain of possible individuals that starts with Talentlessi, continues with Talentlessi+ and then Talentlessi++ (who is slightly more creative than Talentlessi+), etc., and ends with a possible sculptor who is equal to Michelangelo with respect to creativity. Since the first link in the chain is comparable to Mozart in the relevant respect, and since each link is comparable to Mozart in this respect if the preceding link is, it follows that the final link is comparable to Mozart as well. But the final link is equal to Michelangelo with respect to creativity. Thus, Michelangelo and Mozart are comparable with respect to creativity.

These arguments leave open another possibility: the pattern exemplified by the Mozart–Mozart+–Michelangelo case is explained by the fact that the pairs $\langle \text{Michelangelo}, \text{Mozart} \rangle$ and $\langle \text{Michelangelo}, \text{Mozart+} \rangle$ are borderline cases of ‘...more creative than...’, whereas $\langle \text{Mozart+}, \text{Mozart} \rangle$ is not. That is, parity is nothing but the intransitive relation of vague or rough equality. Chang, however, rejects this solution: problems of vagueness, she argues, can be resolved by stipulation; vagueness occurs when the vague concept runs dry and we are left with indeterminacies. But in the Mozart/Michelangelo case the concept does *not* run dry. There is rich conceptual material there to be mined—and introducing the notion of parity helps us to see that. Thus, since (3) counters S-Transitivity*, parity is possible.

Based on this reasoning, Chang further argues that ‘the basic assumptions of standard decision and rational choice theory [are] mistaken: preferring X to Y, preferring Y to X, and being indifferent between them do not span the conceptual space of choice attitudes one can have toward alternatives’ [2002a: 660].¹ This leads her to reject Decision Theoretic Transitivity* [(2b)]:

- (2b) DT-Transitivity*: if A, B, and C are comparable with respect to the relevant value and A is better than B, and C is not better than B, then A is better than C.

Temkin’s argument against S-Transitivity [(1a)] unfolds in the reverse direction: from decision theory to semantics. It is formulated as an attack on the basic axiom of standard decision theory (1b):

¹Chang’s critics agree. Thus, Ryan Wasserman claims: ‘If parity [à la Chang] is possible, the preferences of agents cannot, in general be adequately modelled on standard utility function’ [2004: 392–3]. Cf., however, Gert’s response in Gert [2004] and Rabinowicz [unpublished].

- (1b) DT-Transitivity: If A is better than B and B is better than C, then A is better than C.

Imagine a set of n lives l_1, l_2, \dots, l_n each 2^n years long (n is a very large fixed number), each life containing a period of painful experiences. These lives differ from each other in the duration and intensity of the painful experiences they contain—and in no other respect. In particular, in l_1 , the agent suffers two years of almost unbearable torture. In l_2 , he suffers four years of slightly less intense torture. Generally, for all $0 < i < n$, l_i contains a period of torture of 2^i years long, where the pain of the torture is slightly less intense than that of the torture in l_{i-1} . Following Rachels, Temkin argues for the plausibility of the following:

- (4a) For any painful experience, however intense and long, preferring this experience to one that was only a little less intense but lasted twice as long is rational.
- (4b) There is a finely distinguishable range of painful experiences ranging from mild discomfort to extreme agony.²
- (4c) No matter how long it must be endured, an almost unnoticeable headache is preferable to almost unbearable torture for a significant amount of time.

Thus, it follows, by the first assumption [(4a)], that l_1 is better than l_2 , l_2 is better than l_3, \dots, l_{n-1} is better than l_n ; it follows, by the second assumption [(4b)], that since n is sufficiently large, l_n contains 2^n years of almost unnoticeable headaches; and it follows, by the third assumption [(4c)], that l_n is better than l_1 .

Temkin's decision-theoretic argument can be reformulated as a semantic argument against S-Transitivity [(1a)]. First, (1b) seems as if it is an instance of (1a), the graded adjective being 'good'. The semantics of 'good', however, is complicated and unique; on the basis of Szabó [2001], it might be argued that 'good' is semantically incomplete, and thus cannot substitute Φ in (1a). But assume that Φ stands for '... more painful life ...' or '... contains more pain ...'. Assume further that for any indexes j_1 and j_2 , if it is rationally required to prefer l_{j_1} to l_{j_2} just on the basis of the pain involved in these lives, l_{j_1} is a less painful life than l_{j_2} . Conjoined to these plausible assumptions, Temkin's reasoning entails that

- (5) l_n is more painful than l_{n-1} ,
 l_{n-1} is more painful than l_{n-2}

²As Erik Carlson [2003: 105] noted, (4a) has two different readings. The weaker reading says that for any life l that contains pain of intensity p and duration t [$l(p, t)$] there is $\epsilon > 0$ such that, $l(p, t)$ is better than $l(p - \epsilon, 2t)$. The stronger reading differs with respect to the scope of the quantifiers involved. It says that there is $\epsilon > 0$ such that, for any life $l(p, t)$ (no matter how great p and t are) $l(p, t)$ is better than $l(p - \epsilon, 2t)$. Binmore and Voorhoeve [2003] show that the negation of (4c) does not follow from (4b) conjoined to the weaker reading of (4a); Carlson shows, however, that it does follow from (4b) and (4a)'s stronger reading.

...

l_2 is more painful than l_1 .

And,

(6) l_1 is more painful than l_n .

At least initially, the conjunction of (5) and (6) counters S-Transitivity.

Chang's and Temkin's critiques of standard decision theory won't concern me here. Both DT-Transitivity and (to a lesser extent) DT-Transitivity* were defended by others elsewhere, and the centrality of these axioms and their prima facie attractiveness is recognised by all. I shall hence focus on the threat posed by the Mozart–Mozart+–Michelangelo pattern (as expressed in (3)) to S-Transitivity* [(2a)] and on the threat posed by Temkin's (5) and (6) to S-Transitivity [(1a)].

In the next section I shall sketch the degree analysis of gradability and point to its deep commitment to S-Transitivity and S-Transitivity*. Section III develops a context-sensitive version of the degree analysis which explains Chang's parity and explains away Temkin's intransitivity.

II. The Degree Analysis of Gradability

I shall use Christopher Kennedy's presentation of the standard version of the degree analysis of scalar adjectives [1997, 2001] and then point to some of this analysis' major advantages, in order to illustrate its broad explanatory force. The degree-based account of comparatives and gradable adjectives introduces 'scales' into the ontology. Scales are representations of measurements, that is, sets of objects (the 'degrees') under a total ordering. These objects are individuated by their position on the scale, on one hand, and the 'dimension' along which they are ordered, on the other. Thus, the semantics of gradable adjectives (like 'heavy') and comparatives (like 'heavier than') entails that the relevant property ('heaviness', the dimension) is possessed by (some) entities to different degrees. The standard version of the degree-based semantics treats both adjectives and comparatives as relations between individuals and degrees. In addition, it postulates a measure function as part of their meaning. This postulation requires quantification into the degree argument in the adjective or the comparative.

To illustrate, let the denotation of ' $>_{\text{heaviness}}$ ' be the set of pairs of degrees to which things might be heavy. Then, the logical form of (7) would be (8):

(7) A is heavier than B.

(8) $\exists d_1 \exists d_2 (\text{Heavy}(A, d_1) \ \& \ \text{Heavy}(B, d_2) \ \& \ d_1 >_{\text{heaviness}} d_2)$.

There is only *one* degree to which a thing is heavy (at a time). Hence, assuming a Russellian analysis of definite descriptions, (7) can be read in a more specific way as

- (9) The degree to which A is heavy $>_{\text{heaviness}}$ the degree to which B is heavy.

The logical form of (10) would have (11) as a component:

- (10) A is heavy.
 (11) $\exists d(\text{Heavy}(A,d) \ \& \ d >_{\text{heaviness}} d_{s(\text{heaviness})})$.

The degree denoted by ' $d_{s(\text{heaviness})}$ ' is the degree that identifies a standard for being heavy in the context in which (10)-type sentences are used.³

S-Transitivity [(1a)] and S-Transitivity* [(2a)] follow from construing scales as sets of objects under total ordering:

- (1c) Degrees-Transitivity: For all triples of degrees d_1 , d_2 , and d_3 on a scale ordered along Φ , if $d_1 >_{\Phi\text{-ness}} d_2$ and $d_2 >_{\Phi\text{-ness}} d_3$, then $d_1 >_{\Phi\text{-ness}} d_3$.
 (2c) Degrees-Transitivity*: For all pairs of degrees d_1 and d_2 on a scale ordered along Φ , either $d_1 >_{\Phi\text{-ness}} d_2$, or $d_2 >_{\Phi\text{-ness}} d_1$, or $d_1 = d_2$.

For all triples, d_1 , d_2 , and d_3 , if $d_1 >_{\Phi\text{-ness}} d_2$ and $\text{not}(d_3 >_{\Phi\text{-ness}} d_2)$, then $d_1 >_{\Phi\text{-ness}} d_3$.

These statements are true by virtue of the very concept of degrees.

Admitting degrees that satisfy (1c) and (2c) into the ontology allows for a unified explanation of the transitivity and transitivity* of the great majority of comparatives. Indeed, the fact that 'heavier than' and 'more expensive than' are both transitive and transitive* is fully articulated by (1c) and (2c) and conceived as a feature of the grammar of ' \dots er than' constructions. The simplicity and the universality of this explanation is, I take it, a major advantage of standard degree-based semantics.

Furthermore, the standard analysis enables us to convey the phenomenon of incomparability in a very elegant way. Consider

- (12) Numbers, sets, geometrical points, and every other abstract object have no weight.
 (13) Photons at rest have no weight.

³As I said in the text, this exposition is drawn from Kennedy [2001]. Yet, whereas Kennedy [1997] adopts the nonstandard account, according to which gradable adjectives denote 'measure functions', I adopt the standard degree-based analysis, which treats adjectives as relations between individuals and degrees. Kennedy thinks that quantification over the degree argument of the adjective should be avoided for reasons that have to do with simplicity and parsimony. The linguistic facts mentioned in the texts in relation to (13) – (15) support the quantificational account. See also the next note.

Despite their identical surface structure, the logical form of (12) radically differs from the logical form of (13). For (13) says that the degree to which a photon at rest is heavy is zero, implying that *there is* a degree to which photons at rest are heavy; whereas (12) means that *there is no* degree to which numbers, sets, geometrical points, etc. are heavy. Clearly, the truth of (12) speaks to the intuition that numbers and elephants are incomparable with respect to weight by virtue of the fact that there is *no* degree to which numbers are heavy. (Below [in n. 9]) incomparability will be defined in a more precise way.) Thus, if indeed (9) [The degree to which A is heavy $>_{\text{heaviness}}$ the degree to which B is heavy] is the logical form of (7) [A is heavier than B], then (14) is trivially true, while (15) and its narrow scope negation are false:

(14) Neither is the number seven heavier than George the elephant, nor is George heavier than the number seven, nor are George and seven equal in weight.

(15) George is strictly heavier than the number seven.

([15]'s narrow scope negation) The number seven is strictly heavier than George.⁴

III. In Defence of Transitivity* and Transitivity

The explanation offered by degree-based semantics to the transitivity and transitivity* of most comparatives is simple and straightforward. Of course, Temkin and Chang might reject this explanation: Temkin might dismiss Degree-Transitivity in the light of (5) & (6), and Chang might dismiss Degree-Transitivity* in the light of (3). Still, before moving in this direction, we should ask whether (1c) might be sustained, even if, as Temkin has argued, (5) and (6) are both true, and whether (2c) can be sustained, even if, as Chang has argued, (3) is true. If the answers to these questions are positive, considerations of theoretical parsimony would weigh against rejecting (1c) and (2c).

I shall argue for a positive answer to the second question: Chang's (3) does not counter Degree-Transitivity*. The answer to the first question is more complicated: Temkin's (5) & (6) do not counter Degree-Transitivity; still, the metaphysics of degrees strongly suggests that one of the judgments in Temkin's sequence [(5)] is false. And the context-sensitive version of the degree analysis would clarify *why* the comparisons in (5) *seem* so appealing, even if at least one of them is false.

⁴Compare (15) and its narrow scope negation to 'The current king of France is bald' and 'The current king of France is not bald' under the Russellian analysis of definite descriptions. Note also that, according to Kennedy, comparing the 'incomparable' is linguistically impossible. And this seems to me to be a mistake. See more on incomparability below.

A. Parity as Crude (Rather than Vague/Rough) Equality

The thesis I advance in this subsection is that the comparatives which allow for parity or (what I prefer to call) ‘crude equality’ do that by virtue of being *complex*. I shall show, in particular, that even a relatively simple comparative like ‘balder than’ tolerates the parity pattern because of its mild complexity.⁵ In III(B) I will show that this explanation allows us to sustain Degree-Transitivity*.

Baldness is a complex dimension: the degree to which a subject is bald depends not only on the number of hairs on the subject’s scalp, but also on the distribution of hairs on it. It depends, further, on the relative *weight* of these two factors in determining the exact extent to which a person is bald. As Ryan Wasserman [2004] observes, a man with 2,000 hairs evenly distributed on his scalp (such that no patches of his scalp are totally exposed) is less bald than a person with 3,000 hairs but with all distributed just around the base of his scalp. After all, if the majority of one’s scalp is exposed, one is bald, no matter how many hairs one has on it.

Now, the first crucial fact entailed by the complexity of baldness is this: baldness is a ‘multi-scaled dimension’; there is more than one set of degrees that scale it. To see why this is so, consider the extent to which Larry+ is balder than Larry (Larry is bald to some degree and ‘Larry+’ refers to Larry after he lost an additional hair) as it is described in (16).

(16) Larry is (one hair) balder than he was yesterday.

Obviously, only few baldness-comparisons—those of ‘close’ individuals like Larry and Larry+—proceed by measuring numbers of hairs. Consider ‘remote’ individuals like Barry and Larry. Barry is doing better than Larry with respect to one factor (i.e., Barry has more hairs on his scalp), whereas Larry is doing better than Barry with respect to the other (the width of the totally exposed patches on Larry’s scalp is smaller). To use Wasserman’s example [2004: 396], Larry has 180 hairs while Barry has 200, yet Larry is compensated for his inferiority with respect to the number-of-hairs-factor by his better condition with respect to the width-of-exposed-patches factor. By stipulation, Larry is not definitely balder than Barry, nor is Barry definitely balder than Larry. How, then, do Larry and Barry relate to each other with respect to baldness? There are two possibilities. The pair $\langle \text{Larry}, \text{Barry} \rangle$ is a borderline case of ‘... balder than ...’; alternatively, the advantages Larry and Barry have over each other are cancelled out by each other, such that it is definitely true that Larry and Barry are equally bald.

In any of these cases, Larry is *not* 20 hairs balder than Barry; for, if he were 20 hairs balder than Barry, he would have been, slightly but definitely, balder than Barry. It follows that (16a) is false:

(16a) Larry is twenty hairs balder than Barry.

⁵I have no intention to argue that *every* complex comparative tolerates a parity pattern.

There is no other *positive number* x such that

(16b) Larry is x hairs balder than Barry.

Needless to say, for no x is (16c) true:

(16c) Barry is x hairs balder than Larry.

Conclusion: for all x , neither (16b) nor (16c) is true: there are no number (of hairs) degrees that measure the extent to which Larry is balder than Barry.

Notwithstanding this, it seems clear that the baldness of Larry and the baldness of Barry are *not* incomparable. So, if the degree analysis is adequate, as assumed here, there must be another set of degrees by which this comparison does proceed. Indeed, there must be a scale of *comprehensive* degrees, by which *every* difference in baldness is measurable. We can thus conclude,

(17) MULTIPLICITY: Some complex dimensions (baldness is one of them) are multi-scaled.

Let us now *suppose* that, definitely, Larry is as bald as Barry. Does it follow that Larry+ is balder than Barry? Despite Chang's claim to the contrary, I shall now argue that MULTIPLICITY implies that the answer might be negative. The difference in baldness between Larry/Larry+ and Barry is measurable only by comprehensive degree. That is, Larry (Larry+) and Barry are equally bald by virtue of the fact that the *comprehensive* degree to which they are bald is one and the same. On the other hand, Larry+ is balder than Larry by virtue of the fact that the *number-of-hairs* degree to which Larry+ is bald is greater than the *number-of-hairs* degree to which Larry is bald. Comprehensive degrees belong to one scale; number-of-hairs-degrees to another; they are not ordered on a single scale. And, I wish now to suggest, as a matter of conceptual possibility, comprehensive degrees might be *crude*; Larry and Barry are just *crudely* equal with respect to baldness, because Larry+ and Barry are crudely equal in this respect, as well.

Is my final stipulation (that, definitely, Larry+ is as bald as Barry) consistent with the former ones (that Larry+ is balder than Larry, Larry and Barry are equally bald, and neither \langle Larry+, Barry \rangle nor \langle Larry, Barry \rangle are borderline cases of '... balder than ...')? The answer for which I shall argue in the next few paragraphs is positive—I shall construct a comparative, '... Φ -er than ...', of which all the above stipulations are true. The theory I develop rests on a crucial distinction between vagueness and crudeness which the model I construct in the following paragraphs elicits.⁶

Let Φ -ness be a multi-scaled dimension along which a finite small set of comprehensive degrees $\{D_i\}_{0 \leq i \leq N}$, is ordered. Suppose that if an object is Φ

⁶I use classical logic, so, for simplicity, the vagueness my model generates is epistemic.

to the degree D_0 , it is completely Φ ; if it is Φ to the degree D_1 , it is very, very Φ ; ... if it is Φ to degree D_N it is mildly Φ . Let, $\{d^1_j\}_{0 \leq j \leq N_1}$ and $\{d^2_l\}_{0 \leq l \leq N_2}$ be two sets of noncomprehensive degrees that scale Φ -ness; j ranges over members in the set $\{0, 1/2^{N_1}, 2/2^{N_1}, 3/2^{N_1}, \dots, 1\}$ and l , over natural numbers between 0 and N_2 . N_1 and N_2 are much greater than N . (18) illustrates one type of relation between the different sets of degrees that scale Φ -ness. In this formula, the function f maps the two-dimensional vectors $\langle m, n \rangle$ (where m belongs to $\{0, 1/2^{N_1}, 2/2^{N_1}, 3/2^{N_1}, \dots, 1\}$ and n is a natural number smaller than N_2) to one of the few elements in $\{0, 1, \dots, N\}$. Thus, for every m in $\{0, 1/2^{N_1}, 2/2^{N_1}, 3/2^{N_1}, \dots, 1\}$ and $n \leq N_2$ there is a natural number $r \leq N$ such that $D_r = f(d^1_m, d^2_n)$. More specifically, the comprehensive degrees, $D_0, D_1, \dots, D_{N-1}, D_N$ are associated with a finite increasing series of positive real numbers $K_0 (=0), K_1, \dots, K_{N-1}, K_N$, such that

$$\begin{aligned}
 (18) \quad & f(d^1_m, d^2_n) = D_0, \text{ if and only if, } Wm^2 + (1 - W)n^2 = 0, \text{ and,} \\
 & f(d^1_m, d^2_n) = D_1, \text{ if and only if, } 0 < Wm^2 + (1 - W)n^2 \leq K_1, \text{ and,} \\
 & \dots \\
 & f(d^1_m, d^2_n) = D_N, \text{ if and only if, } K_{N-1} < Wm^2 + (1 - W)n^2 \leq K_N.
 \end{aligned}$$

(W and $1 - W$ [$0 < W < 1$] are fixed constants that represent the weight of the simpler dimensions that constitute Φ -ness in determining the comprehensive degree to which things are Φ .) The crudeness of the comprehensive degrees follows from (19), which is a direct consequence of (18):

$$(19) \quad f(d^1_{m'}, d^2_{n'}) = f(d^1_m, d^2_n), \text{ if and only if the weighted squared distances of } \langle m, n \rangle \text{ and of } \langle m', n' \rangle \text{ from } \langle 0, 0 \rangle \text{ (} \langle d^1_0, d^2_0 \rangle \text{ represents complete } \Phi\text{-ness) is either 0, or between 0 and } K_1, \text{ or between } K_1 \text{ and } K_2, \dots \text{ or between } K_{N-1} \text{ and } K_N.$$

I shall suppose that baldness resembles Φ -ness in all relevant respects. As such, it allows a clear distinction between crude and vague (rough/proximate) equality. Thus, consider borderline cases of ‘balder than’ which my model generates. Consider, that is, the baldness of Harry, Harry+, Curly and Curly+ as represented in Figure 1.

In Figure 1, Harry+ (Curly+) is one hair balder than Harry (Curly), hence non-vaguely balder than Harry (Curly). Still, since the location of the ‘very-very-bald-curve’ is indefinite, it is vague whether they are both very, very bald, or just very bald. If the comparison of the pairs \langle Harry, Curly \rangle , \langle Harry, Curly+ \rangle , \langle Harry+, Curly \rangle , and \langle Harry+, Curly+ \rangle proceeds by comprehensive crude degrees, then it depends *only* on whether they are very, very bald or just very bald. Since this is vague, these four pairs are borderline cases of ‘balder than’.

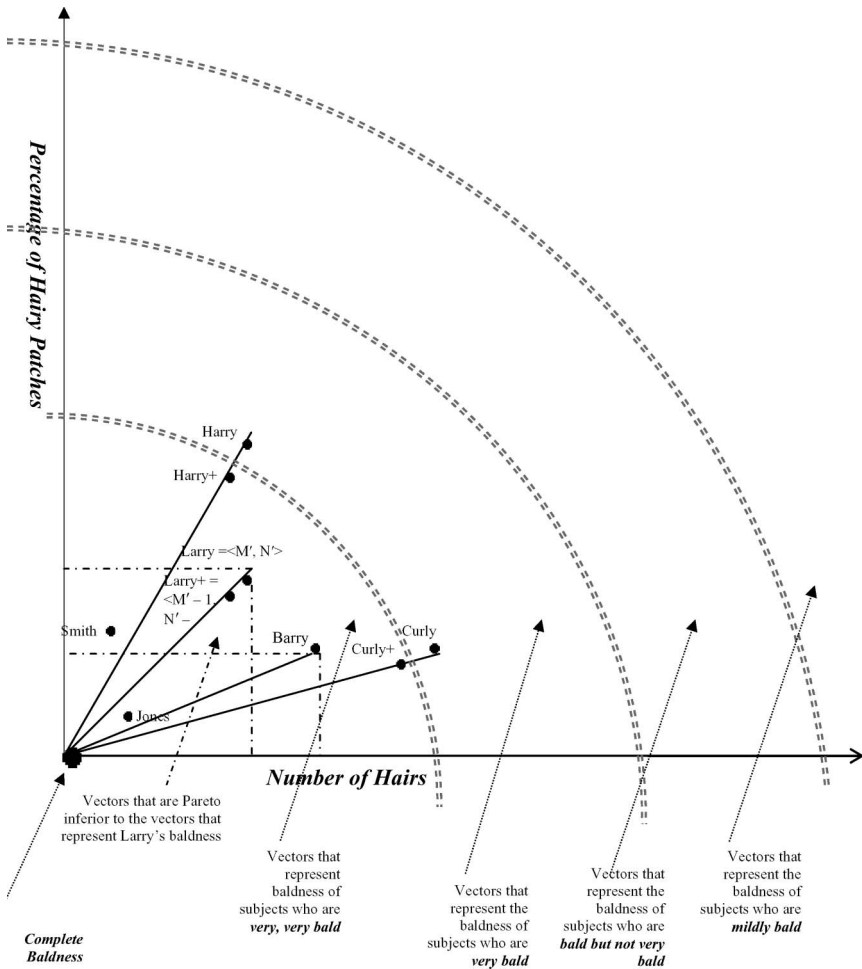


Figure 1. Notes: 1. Read ‘proper name’ as standing for the ‘vector that represents Proper Name’s baldness.’ 2. For simplicity, it is assumed that there are only two factors that constitute baldness, and that their weight is identical.

In contrast, the vectors that represent the baldness of Larry and Barry are much closer to the origin. Larry and Barry are definitely very, very bald. Yet, regarding baldness, Larry and Barry are ‘remote’ individuals, and this means that the comparison of their baldness can proceed by comprehensive degrees only. So, it is definitely true that they are equally bald by virtue of the fact that, although they are not completely bald, both are clearly very, very bald.⁷ And, Larry+ is also very, very bald, so he, too, is as bald as Barry. The relation of crude equality has nothing to do with vagueness:

⁷They would have been (definitely) equally bald had they both been very bald, or bald but not very bald...etc.

Larry and Barry are crudely equal, rather than strictly equal, because Larry+ is definitely balder than Larry.

Generally, the comprehensive degrees that scale Φ -ness are crude if there are objects O1 and O2 that satisfy (20):

- (20) P-PATTERN: O1 and O2 are crudely equal with respect to a complex dimension Φ -ness if (i) $\langle O1, O2 \rangle$ is not a borderline case of ' $\dots \Phi$ -er than \dots ', (ii) the comprehensive degree to which they are Φ is one and the same, (iii) there is an object O2+ such that O2+ is Φ to this comprehensive degree as well, (iv) the difference in Φ -ness between O2/O2+ and O1 is measurable by comprehensive degrees only, yet, (v) the noncomprehensive degree to which O2+ is Φ is greater than the noncomprehensive degree to which O2 is; that is, O2+ is Φ -er than O2.

In other words, if there are complex dimensions of which (21) is true, crude equality (as it is characterized in (20)) is a fourth positive relation.

- (21) CRUDENESS: The comprehensive degrees ordered along Φ -ness are crude.

Now, Chang's Michelangelo/Mozart/Mozart+-example exploits the fact that among the multiple contributory components that constitute artistic creativity, some are measured by cruder degrees than others. Creativity, for example, is constituted (in part) by productivity; if one artist is more productive than another, *and all other things are equal*, the former is more creative than the latter. Mozart+ wrote one piece of music more than Mozart. Hence, by the above logic, Mozart+ is more productive than Mozart by one piece of music: his extra productivity makes him more creative. In other words, the 'productivity degree' to which Mozart+ is creative is slightly greater than the productivity degree to which Mozart is creative.

The set of differences in creativity that are measurable by productivity-degrees is, however, extremely restricted. In particular, since so many things are *not* equal in the comparison of Mozart and Michelangelo, the difference in the creativity of Mozart and Michelangelo is immeasurable by such degrees. And, the comprehensive degree to which Mozart, Mozart+, and Michelangelo are creative is one and the same—regarding creativity, they are all crudely equal; they are all in the same league, somewhere at the very top in their field.⁸

I suggest, in other words, that the phenomenon that Chang tries to capture by the notion of parity is the phenomenon of crude equality. Objects might be (non-vaguely) better, worse, strictly equal, or crudely equal with respect to, say, creativity. They are strictly equal if they are equal with respect to any contributory value; they are crudely equal, if they are equal,

⁸The tension here is obvious: Mozart+ is more creative than Mozart *and* they are crudely equal. I shall dissolve the difficulty in IIIC.

but their comparison can proceed by comprehensive degrees only, and the comprehensive degrees themselves are crude.

There is, of course, a deep difference between the logic of crude equality and the logic of parity as Chang understands it. On the above model, crude (but non-vague) equality is a transitive relation. If it is non-vaguely true that Mozart is as creative as Michelangelo and that Michelangelo is as creative as Mozart+, then the equality at stake is crude, *and* Mozart and Mozart+ are also crudely equal with respect to creativity. Chang's conceptualization of parity resists this entailment. Another difference between her approach and mine is epistemic; on my view, comparing Michelangelo and Mozart is not 'super-hard', as Chang claims, but rather quite easy: each artist is at the very top of his field. This is sufficient for determining that they are equally creative. Now, our ability to compare Michelangelo and Mozart depends on a prior determination that classical music and visual arts are on a par in terms of the creativity it takes to be at the very top of these fields. (Obviously, a top checkers player is not as creative as Michelangelo.) And, of course, establishing how creative one has to be to in order to reach the top of one's particular field is not that easy. Still, for those who are acquainted with the work of these artists, it is easily knowable that they are at the very top of their field, and that this fact entails that they are equally creative.

The above epistemic point suggests that comprehensive comparisons are qualitative. As opposed to differences in quantity, which are divisible and accurately measurable, qualitative differences (or lack of qualitative differences) are crude, such that, *when we confront them*, they are easily visible. Thus, crude degrees divide the relevant space in the way colours divide the spectrum: in terms of quantities—i.e., in terms of wavelengths—the difference between dark red and light red might be greater than the difference between light red and orange. The qualitative difference is, however, greater.

Crude degrees of the type constructed above are everywhere: chess, judo, Olympic games, etc. They share at least three features. First, a person might have to do only slightly better to become an international master in chess, to earn a black belt in judo, or to be qualified for the Olympic games. Second, the *exact* location of the threshold in all these cases is vague—it is determined by convention. Third, once individuals go beyond a threshold by making some small progress, they might have to make greater progress in order to go beyond the next threshold. And, finally, in many cases, the difference between remote individuals regarding a complex dimension is measurable merely by such crude degrees.

B. Contextualizing Scales I: Defending S-Transitivity*

This metaphysical theory of complex dimensions and crude degrees faces important objections. Before presenting and addressing them, I wish to advance a semantic dissolution of Chang's puzzle. I shall argue that the possibility of parity is consistent with S-Transitivity* [(2a)] if it is interpreted as an articulation of Degrees-Transitivity* [(2c)]. Chang denies (2a) on the basis of the truth of sentences whose structure is shared by (22):

- (22) Larry+ is balder than Larry, Barry is not balder than Larry, and yet Larry+ is not balder than Barry.

The parity pattern exemplified in (22), I wish now to argue, does not threaten Degrees-Transitivity* [(2c)].

Consider (22)'s first conjunct:

- (23) Larry+ is balder than Larry.

On the standard versions of degree-based analysis, (23) is, in effect, an existential statement which says that *there are* degrees d_1 and d_2 , such that the degree to which Larry is bald is d_1 , the degree to which Larry+ is bald is d_2 , and $d_2 >_{\text{baldness}} d_1$. In the light of the multiplicity of the scales by which differences in baldness are measured, the truth conditions of (23) are determined only *given a scale to which the degrees d_1 and d_2 belong*. The domain of the existential quantifier in the logical form of (23) should be restricted somehow. The logical form of (23) contains, therefore, a *contextual scale-variable*:

- (24) There are degrees d_1 and d_2 on a scale s such that Bald(Larry, d_1) & Bald(Larry+, d_2) & $d_2 >_{\text{baldness}} d_1$.

In the usual context, the proposition expressed by (24) is that the *number degree* to which Larry+ is bald is greater than the number degree to which Larry is bald. The value of s in this context is the scale of number-of-hairs degrees.

Now, consider the logical form of the other two conjuncts in (22):

- (25) There are degrees d_1 , d_2 , and d_3 on a scale s such that Bald(Larry, d_1) & Bald(Larry+, d_2) & Bald(Barry, d_3) & Not($d_1 >_{\text{baldness}} d_3$) & Not($d_2 >_{\text{baldness}} d_3$).

Again, in (25), the variable 's' is a contextual scale-variable. But the only scale by which the (25)-comparisons can proceed is of comprehensive degrees of baldness. That is to say, in (22), the difference between Larry and Larry+ is measured by number-of-hairs degrees while the differences between Barry and Larry+/Larry are measured by comprehensive degrees.

It is by now clear, I hope, why (22) does not counter Degrees-Transitivity* [(2c)] and (hence) S-Transitivity* [(2a)]. Recall, in the degree analysis, S-Transitivity* follows from constructing scales as sets of objects under a total ordering. Hence, the degree analysis predicts (merely) that *if all the following comparisons proceed by the same scale*, if A is Φ -er than B, and C is not Φ -er than B, then A is Φ -er than C. (22), however, involves comparisons that proceed on *different* scales. Thus, in effect, Chang is right: ' $\dots\Phi$ -er \dots ' constructions are not necessarily transitive*; but this has nothing to do with the individuation of degrees as it is described in (2c). The same idea can be easily applied to Chang's own example of artistic creativity. She is right that (26) [formerly (3)] rings true:

- (26) Mozart+ is more creative than Mozart, and Michelangelo is not more creative than Mozart. Still, Mozart+ is not more creative than Michelangelo.

If (26) involves different scales, it leaves Degree-Transitivity* intact. The difference between Mozart and Mozart+ is measured by productivity degrees, whereas the comparisons of Mozart and Michelangelo and of Mozart+ and Michelangelo proceed by comprehensive degrees. Indeed, (26) leaves (2c) and hence (2a) intact.⁹

C. Objections and Replies

1. The most basic objection to the theory I have just developed reads as follows. Mozart+ is more creative than Mozart. Yet, if the comprehensive degree to which Mozart and Mozart+ are creative is one and the same, Mozart and Mozart+ are equally creative. The theory thus entails a flat contradiction. The same is true of Larry+ and Larry: they are equally bald (as the crude degree to which they are bald is one and the same), and yet the former is balder than the latter (as the number-of-hairs degree to which Larry+ is bald is greater than the number-of-hairs degree to which Larry is bald).

The context-sensitive semantics sketched in the previous subsection, however, easily dissolves this worry. In the usual contexts in which Mozart and Mozart+ or Larry and Larry+ are compared, the sets of non-crude divisible degrees have a special status. In such ‘all-other-things-being-equal-comparisons’ of close individuals, we attend to the small quantitative, measurable differences between the compared objects. Hence, asserting of Mozart and Mozart+ that they are equally creative, or of Larry and Larry+ that they are equally bald, is (usually) false. For the salient scales in the contexts of comparing these close individuals are, respectively, the set of productivity degrees and the set of number-of-hairs degrees. And it is obviously false that the productivity degree to which Mozart and Mozart+ are creative is one and the same, and that the number-of-hairs degree to which Larry+ and Larry are bald is one and the same.¹⁰

2. A second difficulty is that the fact that Mozart and Mozart+ are equal in all respects except productivity, and the fact that in this respect Mozart+ is doing slightly better than Mozart, seem to entail that Mozart+ is more creative than Mozart, *all relevant aspects included*, i.e., *as measured by comprehensive degrees*.

⁹Note that my version of the degree analysis constructs partial orders as a form of incomparability: O1 and O2 are comparable regarding Φ -ness only if there are two degrees d1 and d2 such that O1 is Φ to a degree d1, O2 is Φ to a degree d2, and d1 and d2 belong to the same scale. Note further that there might be multiscaled but simple dimensions. For example, some writers argue that outcomes are ordered with respect to equality or utility only if the population involved in these outcomes is fixed (see Temkin [1987]).

¹⁰The fact that we tend to ignore or underweight similar or identical dimensions of many-dimensional things when we directly compare them is supported by the psychological literature on similarity-based decision-making. Like this paper, the literature on similarity-based decision-making suggests that our method of comparing multidimensional things is context-dependent: when we compare people like Larry and Larry+ we use a different method of comparison than when we compare Larry (or Larry+) and Barry. Roughly, this literature suggests that in comparing Larry and Larry+, we ignore the dimension along which they are identical and focus only on the dimension along which they are different; whereas when we compare Larry (or Larry+) and Barry, we may engage in some overall assessment of the contribution that both dimensions (number of hairs and percentage of bald patches) make to the baldness of each. See Tversky [1969]; Bar-Hillel and Margalit [1988]; Binmore and Voorhoeve [2006]; Slovic and MacPhillamy [1974]; Tversky and Kahneman [1986]; and Sen [1993].

In order to address this objection, I need to point to a fundamental feature of ‘all-else-being-equal’ comparisons. Observe that comparing the creativity of Mozart and Mozart+ solely by (non-crude) productivity degrees is possible because Mozart and Mozart+ are identical in all other respects. Or, more generally, close individuals can be compared in a non-crude way because the *weight* of the productivity factor (that determines the comprehensive, crude, degree to which these individuals are creative) can be ignored. I thus suggest a distinction between *all-aspects-included-comparisons*, which involve weighing the different factors against each other and *all-other-things-being-equal* comparisons: noncomprehensive, one-aspect comparisons are non-crude, because they are insensitive to the weight of the factor by which they proceed.

The above worry might be put in a slightly different way: Mozart and Mozart+ are equal in all respects except productivity, while in this respect Mozart+ is doing slightly better than Mozart. It follows—the objector argues—that Mozart+ is better than Mozart, *all things considered*; and this implies that the comprehensive degrees to which Mozart and Mozart+ are creative are not the same.

I insist, however, that the comparison expressed by the sentence ‘Mozart+ is more creative than Mozart, *all things considered*’ is *not* comprehensive. The modifier (‘all things considered’) does not fix the comprehensive scale as the scale by which *all* ‘all-things-considered comparisons’ proceed. To see why, notice that the sentence ‘Mozart+ is more creative than Mozart, but he is not more creative than him, *all things considered*’ is semantically anomalous. This anomaly suggests that comparisons in general—and in particular, ‘other things being equal’ comparisons—are ‘all-things-considered’ comparisons as well (unless it is explicitly stated otherwise). Now, degrees of creativity are comprehensive in the sense that they measure *all* differences in creativity, by virtue of being sensitive to the weight of *all* contributory components of the dimension they scale. In the Mozart/Mozart+ case, the *weight* of the productivity component in determining the comprehensive degrees to which Mozart and Mozart+ are creative is *not* one of the things that ought to be considered.

What, then, is the role of the modifier ‘all-things-considered’? What does it contribute to the logical form of ‘Mozart+ is more creative than Mozart, all things considered’? This sentence means, I suggest, that there are degrees d_1 and d_2 on a scale s such that $\text{Creative}(\text{Mozart}, d_1) \ \& \ \text{Creative}(\text{Mozart}+, d_2) \ \& \ d_2 >_{\text{creativity}} d_1$ & s is the appropriate set of degrees for measuring this difference.

One important objection to this suggestion merits close attention. The objection is based on a seductive analogy between the Mozart/Mozart+ case and ‘the equal-length rectangles’ case. If you want to determine which of two rectangles that have an equal length is larger, there is no need to multiply the length and width of each and compare their area. Still, the *weight of the length factor* is not irrelevant: the area of the rectangle is a function of both factors. Similarly for the Mozart/Mozart+ case: the relative weight of the various relevant factors might well be relevant; since these factors are all equal, we are warranted in ignoring their weight when

making our determination. In no sense, the objection goes, is their weight irrelevant.

We should, I believe, resist the analogy. The difference between largeness of rectangles and creativity/baldness of individuals can be brought out by two facts. First, the latter dimensions are multi-scaled: when close individuals are compared, baldness/creativity is measured by number-of-hairs/productivity degrees. Thus: [(14)] Larry is one hair balder than he was yesterday; Mozart+ is more creative than Mozart by one piece of music. In contrast, there are no separate width degrees by which we measure the largeness of rectangles; in no sense are the equal-length rectangles ‘close’ while other rectangles are ‘remote.’ Relatedly, the degrees by which we measure largeness of geometrical objects are never *crude*; the weight of the different factors that constitutes this dimension is fine-tuned. In contrast, we did not bother to develop a comprehensive scale of more divisible degrees of baldness—such a fine-tuned scale is useless. And, owing to the very nature of creativity, it seems impossible to create fine-tuned comprehensive degrees of creativity. Non-crude comparisons of close individuals (Larry and Larry+/Mozart and Mozart+) are possible because, in comparing them, we can do without the weight of the only differentiating factor.

In sum, given the weight of the productivity factor, the small difference between Mozart and Mozart+ in productivity, and the crudeness of comprehensive comparisons of creativity, Mozart and Mozart+ might well be creative to the same *comprehensive* degree. So they are crudely equal in this respect.

3. Suppose that there are three simpler factors that constitute a complex dimension Φ' . Consider objects that differ in the first two factors while being strictly identical with respect to the third. By the logic of the theory defended here, there is a set of (relatively) comprehensive degrees that measure differences in the Φ' -ness of these objects. And, there would be *another* set of comprehensive degrees that measure differences in Φ' -ness among objects that differ in the last two factors but are strictly identical with respect to the first. And so on. The most comprehensive degrees will measure differences in Φ' -ness between every two objects that differ in all three factors. Initially, such proliferation of comprehensive degrees might seem very implausible.

I agree that, according to the theory I defend here, there might be a proliferation of comprehensive degrees. However, I suggest that we might be uninterested in many of them; we might measure some differences by crude degrees even where a finer comparison is possible. Moreover, we should recognize that, to an extent, it is simply a fact that comprehensive scales do proliferate. In order to compare the creativity of Bach and Mozart (who had radically different styles of composing), we need a set of more comprehensive degrees than the set of degrees we need in order to compare the creativity of composers that had the same style. The degrees by which Mozart and Bach are compared would be less comprehensive than the degrees by which Mozart and Michelangelo are compared. We shall need even more comprehensive degrees in order to compare the creativity of Mozart and Plato.

4. The final challenge concerns the extent to which Jones and Smith (of Figure 1) are balder than Larry and Barry. Compare, first, Jones’s baldness

to Larry's. This comparison is not an all-other-things-being-equal comparison, for Jones does worse than Larry regarding *both* factors that constitute baldness. Therefore, it might be thought that my analysis entails that their comparison proceeds by comprehensive degrees; that the only thing that can be said of Jones and Larry is that they are bald to the same crude degree. This—the objector justly complains—is very counterintuitive; Jones is balder than Larry if he does worse than Larry with respect to all factors that constitute baldness.

I deny the premise of this objection. Suppose that Larry has 180 hairs on his scalp while Jones has only 100. As Larry is doing better than Jones also with respect to the width-factor, the extent to which Jones is balder than Larry *is* measurable by number-of-hairs degrees. Indeed, Jones is 80 hairs balder than Larry. Moreover, I suggest that in saying 'Jones is balder than Larry' (without specifying a particular scale) we express a true proposition. For the logical form of this sentence is given by (27):

- (27) There are degrees d_1 and d_2 on a scale s such that $\text{Bald}(\text{Larry}, d_1) \ \& \ \text{Bald}(\text{Jones}, d_2) \ \& \ d_2 >_{\text{baldness}} d_1$.

And, in the usual context, the value of the contextual variable 's' in (27) is undetermined: it might be either the set of number-of-hairs degrees or the set of width-of-exposed-patches degrees. That is, the *context does not make any one of these non-crude scales salient*. And yet, in the context of comparing such individuals, we do not use the comprehensive scale that we use in comparing remote individuals.

Consider, in the light of this suggestion, the comparison of Larry, Barry, and Smith. Larry and Barry are equally bald: the comprehensive crude degree to which they are bald is one and the same. The same is true of Barry and Smith. Yet, Larry does better than Smith with respect to both factors. So my above suggestion implies that Smith is balder than Larry. Larry, Smith, and Barry generate a parity pattern of their own, just as Larry, Larry+, and Barry do.

D. Contextualizing Scales II: Defending S-Transitivity

Temkin's judgments regarding the painfulness of $l_1, l_2, l_3, \dots, l_n$ are not inconsistent with the degree analysis as I have elaborated it. Even so, given the nature of the degrees to which lives might be painful, and the relationships among the various sets of degrees that scale the dimension of life-painfulness, Temkin's judgments are likely to be false. Since the degree analysis seems to me plausible, I tend to believe that one of Temkin's judgments *is* false. I won't, however, try to argue for the view I am inclined to adopt. Instead, I shall show that the degree analysis can clarify why some of Temkin's judgments *look* true, even if they are false. Temkin's mistakes (if there are any) result from a natural delusion.

Consider again $\{l_1, l_2\}, \{l_2, l_3\}, \dots, \{l_{n-1}, l_n\}$; Temkin argues for (28) [formerly, (5)]:

- (28) l_n is more painful than l_{n-1} .
 l_{n-1} is more painful than l_{n-2} .
 ...
 l_2 is more painful than l_1 .

He additionally argues for (29) [formerly, (6)], and claims that its negation follows from (28) by S-Transitivity.

- (29) l_1 is more painful than l_n .

The first ingredient of my defence of S-Transitivity comes down to this: despite appearances to the contrary, the negation of (29) does not follow from (28) by Degrees-Transitivity [(1c)] *alone*. This is because the comparisons in the (28) sequence proceed by *non-comprehensive* degrees, while (29) proceeds by *comprehensive* degrees. I suggest, in other words, that the conjunction of (28) and (29) would have been less seductive had accepting it involved a flat denial of Degrees-Transitivity.

To see why the negation of (29) does not follow from (28) by Degrees-Transitivity, we should pay attention to three facts.

First, life-painfulness is a complex dimension, scaled by two sets of noncomprehensive degrees, namely: duration-degrees and intensity-degrees. Consider $l(p, t)$ —a life that contains an experience of pain of duration t and of intensity p . Intuitively, $l(p, 2t)$ is (more or less) twice as painful as $l(p, t)$. Suppose, additionally, that Jones suffers from a painful terminal disease. Jones's physician recommends a drug, saying 'the remainder of your life will be approximately 20% less painful, thanks to the drug': the doctor says, in effect, that approximately, $l(.8p, t)$ is 20% less painful than $l(p, t)$. This suggests that the comprehensive degree to which a life is painful is determined by at least three factors: the intensity of the pain this life contains, its duration, and the relative weight of these two factors.

Second, I suggest that (28)-comparisons proceed by noncomprehensive degrees, and, more specifically, that they are 'dominated' by duration degrees. For these comparisons result from the following generalization: for all i , if the painful experience in l_{i+1} is twice as long as the painful experience in l_i —and *all other things are almost equal*— l_{i+1} is approximately twice as painful as l_i . That is, l_{i+1} is approximately twice as painful as l_i by virtue of two facts: (a) the painful experience in l_{i+1} is twice as long as the painful experience in l_i , and (b) the painful experience in l_i is only *slightly* more intense than in l_{i+1} . So, the (28)-comparisons are true by virtue of the seeming self-evidence of (30):

- (30) l_n is approximately twice as painful as l_{n-1} .
 l_{n-1} is approximately twice as painful as l_{n-2} .

...

l_2 is approximately twice as painful as l_1 .

As the term ‘twice as painful’ clearly suggests, the comparisons in (30) do not proceed by comprehensive *crude* degrees; they proceed, in effect, by duration degrees. Though the intensity factor is *not* ignored or disregarded, it is marginalized; the modifier ‘approximately’ is merely a reminder that facts about the duration of the painful experience do not tell the whole story. I shall thus say that the comparisons in (28) and in (30) are *dominated* by duration degrees and *marginalize* the intensity factor.

Third, (29) proceeds by comprehensive degrees; as other things are unequal (rather than almost equal), the difference in the painfulness of l_1 and l_n cannot be measured by noncomprehensive degrees. So the comparison of such lives must be comprehensive, and (consequently) crude.

If my above claims are true, (29)’s negation does not follow from (30) via Degrees-Transitivity alone; and, if the relations between (28) and (30) are as I describe them, (29)’s negation does not follow from (28) via Degrees-Transitivity alone, as well. So, by committing ourselves to the conjunction of (28) and (29) we do not deny Degrees-Transitivity [(1c)] and hence S-Transitivity [(1a)]. This partly explains why the conjunction of (28) and (29) might seem appealing, and concludes the first element of the resolution of the Rachels-Temkin paradox advanced here.

Now, (29)’s negation does follow from (30) via Degrees-Transitivity if (31) is true:

- (31) For all i ($0 \leq i < n$), if l_{i+1} is approximately twice as painful as l_i , it is not the case that, crudely, l_{i+1} is as painful as l_i .

Is (31) true? It certainly looks true.¹¹ Simply: l_i and l_{i+1} substantially differ with respect to the durations of the painful experiences they contain. With respect to all other factors of life-painfulness, they are nearly equal. As the difference in duration is substantial, l_i and l_{i+1} cannot be *crudely equal* with respect to how painful they are. If so, rescuing Degrees-Transitivity is possible only if at least one of the clauses in (30) is false.

I shall now use the degree-based semantics developed here to explain why, despite appearances to the contrary, one of the comparisons in (28) might be false. To do that, I should like to develop a more detailed account of the way the factors of duration (of the pain) and (its) intensity interact in determining the comprehensive degree to which lives are painful. The simplest model is multiplicative: it says that the degree to which $l(p, t)$ is painful equals pt . The multiplicative model, however, fails. Based on a wide

¹¹Interestingly, Parfit might be read as denying (31). He claims that one’s reason to save oneself rather than a stranger is *crudely* as strong as one’s reason to save one stranger rather than oneself. Parfit also argues that one’s reason to save ten strangers is ten times as strong as one’s reason to save only one stranger. Surprisingly, Parfit thinks that these assertions are consistent with the following: one’s reason to save oneself rather than *ten* strangers is, crudely, as strong as one’s reason to save ten strangers rather than oneself. See Parfit forthcoming, chapter 5, section 15.

range of systematic empirical research, psychologists argue that the duration of pains is sharply underweighted when the painfulness of an experience is retrospectively assessed. More specifically, they show that patients retrospectively judge a day that contained a short (t_S -long) experience of intense pain (p_I) to have been more painful than a day that contained a long (t_L -long) experience of mild pain (p_M). This would be their judgment even if it is clear that $t_{SP_I} = t_{LP_M}$ [see a survey and further evidence in Ariely and Carmon 2000].

Temkin makes a suggestive remark that might explain this phenomenon:

... torture's badness might range from 0 to 10, depending on its duration, with two years of torture being, say, a 7. A hangnail's badness might range from 0 to 1. Prolonging a hangnail increases the value of the decimal places representing its 'badness score,' but the fundamental gap between 1 and 7 is never affected.

[1996: 92]

That is, there are levels of pain-intensity p_M (very mild pain) and p_I (very intensive pain) such that $l(p_M, nt)$ is less painful than $l(p_I, t)$, however large n is. This, of course, counters the multiplicative model—it suggests that intensity matters *more* than duration in determining the overall painfulness of lives.

The failure of the multiplicative model does not undermine the relationship between (28) and (30): if all other things are *almost* equal—i.e., if ε is *small enough*— $l(p - \varepsilon, 2t)$ is approximately twice as painful as $l(p, t)$. Furthermore, the tendency to marginalize the intensity factor is strengthened because of the difficulty to assess the comprehensive painfulness of each alternative life, taken by itself. To repeat a point I made above, the (28) sequence is based on the (30) sequence, and the (30) comparisons focus on the dissimilar dimension—the duration of the pain—while marginalizing the intensity of this pain.

How small should ε be to allow marginalization of the intensity factor? This depends, I suggest, on the weight of the intensity factor in determining the *comprehensive* degree to which a life is painful. Suppose, then, that there is a level of pain intensity p_{M^*} such that a pain of intensity p_{M^*} is *very mild*, whereas for all $\varepsilon > 0$, a pain of intensity $p_{M^*} + \varepsilon$ is mild but not very mild. There is, in other words, a qualitative difference between pains of intensity $p_{M^*} + \varepsilon$ and pains of intensity p_{M^*} . And suppose that the difference between very mild pains (pains whose degree of intensity $\leq p_{M^*}$) and mild (but not very mild) pains (pains whose degree of intensity $> p_{M^*}$) is never small enough to support a similarity-based comparison of $< l(p_{M^*} + \varepsilon, t), l(p_{M^*}, 2t) >$. If so, somewhere in the (30) sequence, duration degrees are used in order to measure a difference that is measurable solely by comprehensive degrees. Now, given its relation to the (30) sequence, this misuse of duration degrees is likely to lead to a mistake in the (28) sequence as well. After all, in comparing two alternatives that each has two dimensions, the marginalization of the similar dimension involves underweighting its contribution.

In (30), and hence in (28), we *systematically* underweight the contribution of the intensity factor, so, at a certain point, this might lead us astray.

Still, as wholes, the (30) and (28) sequences generate a natural delusion. In comparing the painfulness of lives we can marginalize the difference between pains of intensity p_I and pains of intensity $p_I - \varepsilon$, between pains of intensity $p_I - \varepsilon$ and pains of intensity $p_I - 2\varepsilon, \dots$ etc. But we cannot marginalize the difference between pains of intensity p_{M^*} and pains of intensity $p_{M^*} + \varepsilon$. This is very confusing: up to a point, the comparison of $\langle l(p + \varepsilon, t), l(p, 2t) \rangle$ can be dominated by duration-degrees, whereas suddenly—at one point—this type of comparison fails.¹²

Conclusion

From a semantic standpoint, Chang's argument for the possibility of parity and the reasoning that leads Temkin to doubt S-Transitivity are very important. They bring out overlooked features of some complex dimensions. First, a complex dimension Φ -ness might be multi-scaled by virtue of its complexity. Second, comparisons of (multi-scaled) Φ -ness are context-sensitive. Finally, in many contexts, how Φ something is can be a 'crude' (but not vague) judgement; several different combinations of simpler attributes translate into the same judgement of Φ -ness. These features fully explain the phenomena which Chang and Temkin discovered. I have shown, moreover, that, despite Chang's and Temkin's claims to the contrary, the facts to which they draw our attention should not shake our confidence in S-Transitivity or in S-Transitivity*.¹³

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¹²The above argument is indebted to Norcross [1997]; and Voorhoeve [forthcoming].

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